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Human memory retrieval as Lévy foraging

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Abstract

When people attempt to recall as many words as possible from a specific category (e.g., animal names) their retrievals occur sporadically over an extended temporal period. Retrievals decline as recall progresses, but short retrieval bursts can occur even after tens of minutes of performing the task. To date, efforts to gain insight into the nature of retrieval from this fundamental phenomenon of semantic memory have focused primarily upon the exponential growth rate of cumulative recall. Here we focus upon the time intervals between retrievals. We expected and found that, for each participant in our experiment, these intervals conformed to a Lévy distribution suggesting that the Lévy flight dynamics that characterize foraging behavior may also characterize retrieval from semantic memory. The closer the exponent on the inverse square power-law distribution of retrieval intervals approximated the optimal foraging value of 2, the more efficient was the retrieval. At an abstract dynamical level, foraging for particular foods in one's niche and searching for particular words in one's memory must be similar processes if particular foods and particular words are randomly and sparsely located in their respective spaces at sites that are not known a priori. We discuss whether Lévy dynamics imply that memory processes, like foraging, are optimized in an ecological way.
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Many diffusion phenomena in physical and chemical settings are describable as object motions of dependent steps that can take very different magnitudes of time to perform [1]. The motions in question satisfy a Lévy probability density distribution defined by

$$P(t_j) \sim (t_j)^{-\mu}, \tag{1}$$

where $t_j > 0$ is the interval length, and $1 < \mu \leqslant 3$. Distributions with $\mu \geqslant 3$ are Gaussian and distributions with $\mu \leqslant 1$ are distributions that cannot be normalized [2]. The ubiquity of the so-called anomalous diffusion expressed by Eq. (1) has encouraged the question of whether such phenomena might also occur in biology [3]. Evidence that cellular and animal behavior exhibit anomalous diffusion [1] invites inquiry into the biological scope of Eq. (1). Does it, for example, characterize aspects of a biological cognitive function such as memory?

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Table 1
Retrieval performance for individual participants together with $|\mu|$ estimates and their r^2 values for the three plotting procedures applied to the data with trend (upper pairs of numbers) and without trend (lower pairs of numbers)

Participant	Retrieved	Repetitions	Efficiency	LT		CD		LBN	
1	143	1	.15	1.58	.92	1.81	.89	1.71	.94
				1.43	.92	1.85	.86	1.75	.89
2	311	4	.24	2.07	.92	2.00	.92	1.98	.96
				1.70	.94	2.08	.89	2.21	.91
3	185	4	.21	2.02	.91	2.03	.96	1.79	.99
				1.85	.89	2.13	.93	1.71	.97
4	147	2	.17	1.68	.94	1.88	.97	1.74	.99
				1.78	.87	1.90	.95	1.73	.97
5	141	3	.11	1.57	.88	1.76	.93	1.49	.98
				1.28	.88	1.88	.86	1.50	.91
6	64	3	.12	1.45	.87	1.82	.93	1.42	.99
				1.21	.98	1.83	.89	1.74	.92
7	110	8	.09	1.24	.96	1.67	.88	1.37	.88
				.98	.92	1.69	.83	1.06	.97
8	136	3	.15	1.64	.89	1.84	.86	1.43	.87
				1.28	.80	1.90	.82	1.82	.85

We conducted a memory experiment along the lines pioneered by Bousfield and Sedgewick [4]. Eight participants (average age 29.3 years) were asked to recall verbally as many animal names as possible, without repetition, within an approximately 20 min span (Table 1). The free recall task was performed while seated comfortably in front of a stand-mounted microphone connected to a laptop computer via a USB preamp. Participants were asked to give only appropriate responses, and minimize extraneous noises and comments. Responses were recorded continuously in 44.1 kHz mono using the audio recording and processing software Audacity. For each of the sound files, the utterances (the retrieved words) were isolated and their onset and offset times determined. The variable of primary interest was the interval between utterances, the inter-retrieval interval, IRI (Fig. 1a). The total number of names recalled, and the number of recalls that were repeats of a prior recall, are shown in Table 1. Post-experiment debriefing suggested that participants tended not to notice a repetition as such.

The original investigation [4] suggested that the exponential function relating cumulative recall to time (Fig. 1b) could be modeled as a process of sampling with replacement. As items are sampled from the pool of all possible items satisfying the specified category, the probability of sampling a novel item decreases and the time required to retrieve a novel item thus increases. Most of the research on freely recalling category members has been directed at refinements of this model and its parameters [5]. Sampling with replacement is limited, however, by the fact that it predicts a smooth curve and is thus not well suited to accommodating the key observations that successive retrievals tend to occur intermittently in bursts (Fig. 1a) and that the bursts often involve clusters (e.g., chinchilla, skunk, mink, beaver; from Ref. [4]).

The strategy of addressing the sequence of retrievals as a series of IRIs is confronted by the sequence's two non-stationary properties expressed in Fig. 1a (short and long intervals interspersed) and Fig. 1b (exponential cumulative growth). Both non-stationary properties are problematic for most standard time-series analyses, whether linear or nonlinear. Although clustering has been investigated as a phenomenon in its own right using modern forms of time series analysis, the research in most cases has been restricted to either the series of clusters or the cluster as a series [5].

At this point, no model has been able to predict successfully the exponential increase in the interval between retrievals interrupted by bursts of shorter retrieval intervals. The ubiquity and systematic nature of this
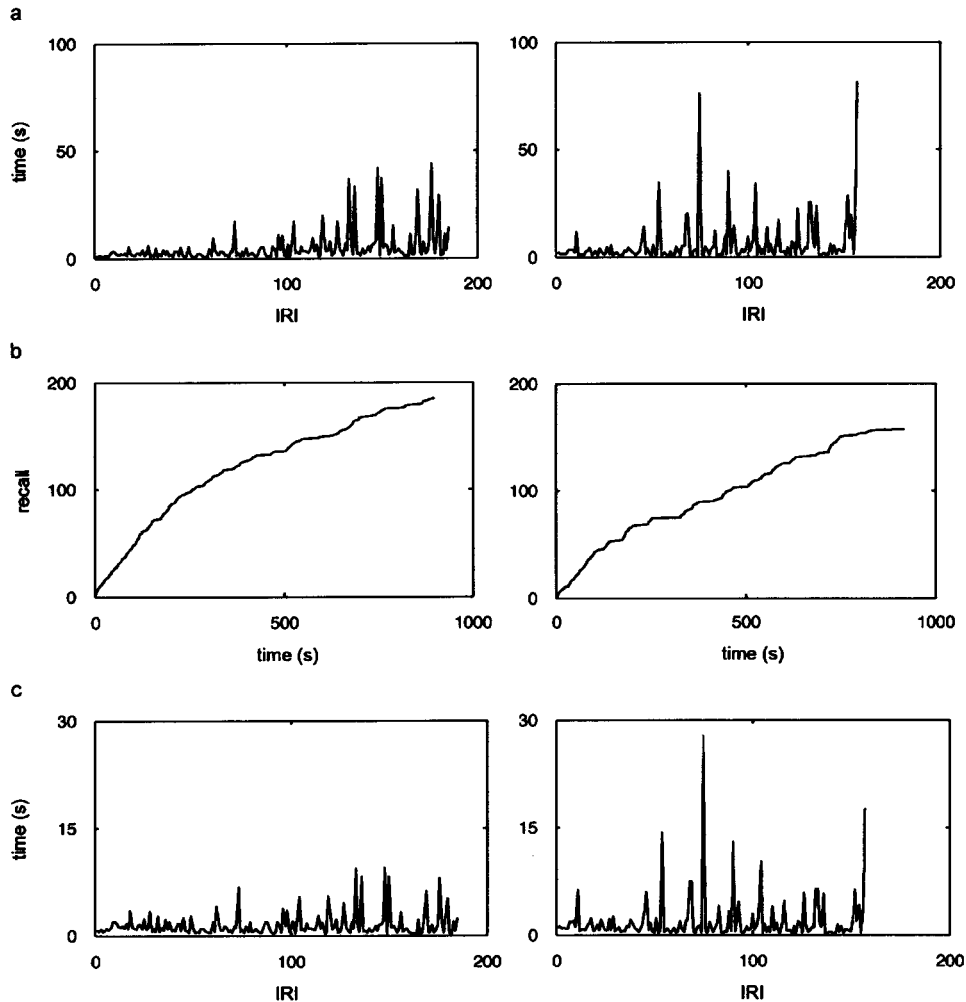
Fig. 1. (a) The IRI in seconds between successive retrievals plotted as a function of position in the sequence of IRIs for two participants (3 and 4 in Table 1). (b) The IRI sequences of (a) are plotted in the standard format as the negatively accelerated growth of the cumulative recall over the time course of recall. (c) The IRI sequences of (a) are plotted following subtraction of the exponential growth depicted in (b).

pattern of retrieval intervals suggests a larger, encompassing principle. Most specifically, it suggests that the pattern should be viewed holistically as a unitary process rather than as the sum of two non-stationary processes. Because of the difficulties and limitations inherent in describing the pattern of retrieval intervals as a time series, the expected encompassing principle must be sought through an analytic method that does not rely on order information.

Examination of the distributional properties of the temporal intervals is one such method. Although a distributional description explicitly removes sequential order, the inherently relational nature of IRIs preserves the temporal aspect of the data. Given the pattern of retrieval intervals, we should expect a distribution capable of generating some indefinitely long intervals but dominated by shorter intervals (Fig. 1a). Indefinitely long intervals can only be produced by a distribution with an infinite tail—a requirement that suggests the Lévy distribution of Eq. (1) rather than a Gaussian distribution [2].

As noted, Eq. (1) characterizes a variety of biological phenomena. Of special significance to memory retrieval are the indications that it characterizes the foraging patterns of different animal species at different phyletic levels [6–10]. Actual and simulated foraging intervals (the times between finding a resource in non-destructive foraging) are fit by a Lévy distribution and optimized when $\mu = 2$ [11–13]. At an abstract

dynamical level, foraging for foods of a particular type and searching for words of a particular type must be processes of like kind if particular foods and particular words are randomly and sparsely located in their respective spaces (niche, memory) at sites that are not previously known.

Accordingly, if the dynamics of free semantic recall are the dynamics of foraging, meaning that the intervals between successive memory retrievals form a Lévy distribution, then the following predictions should hold: (1) for each individual participant, IRIs should abide a power law with a slope between $\mu = 1$ and 3; (2) the more efficient the retrieval the more closely should μ approximate 2; and (3) a plot of the intervals between successive retrievals using a Lévy walk model should reproduce the characteristic pattern of Lévy flights [6,11].

We found strong support for Prediction 1 (see Table 1 and Fig. 2). In searching for animal names, each of the eight participants behaved according to the dynamics of a Lévy forager in an environment of sparse target sites that can be revisited. Three methods of increasing sophistication and accuracy, namely, logarithmic transform (LT), cumulative distribution (CD), and normalized logarithmic binning (LBN) [14–16], were used to estimate μ. All three methods plot the data in double log 10 coordinates. LT plots a histogram of frequency against a constant bin width. LBN plots the histogram against logarithmically increasing bins (each bin k increased by 2^k) and uses the geometric midpoints to plot the data. The normalization procedure determines the (logarithmically) central value for each bin by estimating its probability density $f(n_k) = (1/2^k)(i_k/I)$ where i_k is the frequency of IRIs in bin k, I is the total number of IRIs and 2^k is the width of the bin [16]. CD is a binless method familiarized by the rank/frequency plot [14,15]. IRIs were sorted in increasing order of size with IRI rank plotted as a function of IRI size (largest IRI = Rank 1). In LT and LBN the slope corresponds to $-\mu$; in CD the slope is $1-\mu$. In respect to estimating μ, LBN is most accurate and LT is least accurate [14]. For
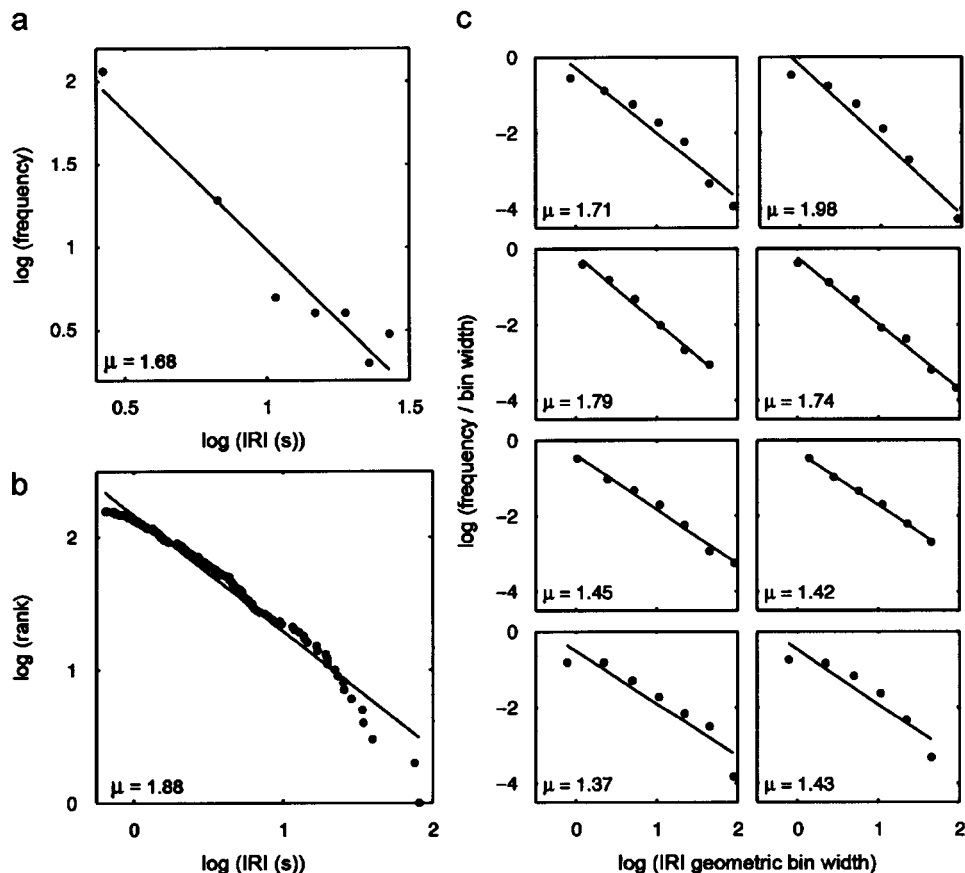

Fig. 2. The three methods used for determining power law behavior and μ are depicted in (a) and (b) for Participant 4 and in (c) for all participants. The three methods described in the text were (a) logarithmic transform (LT), (b) cumulative distribution (CD), and (c) logarithmic binning with normalization (LBN).

each of the three methods, the outcome was $1 < \mu \leqslant 3$. (All linear regressions summarized in **Table 1** and **Fig. 2** were significant at $p < .05$, the majority at $p < .001$.)

To address the possibility that evidence for Prediction 1 is an artifact of the exponential cumulative growth (Fig. 1b), the analyses were redone following detrending. The exponential best fit of each participant's IRI series was determined and subtracted from the series. Examples of the fluctuations in recall intervals after correcting for the exponential growth are shown in **Fig. 1c**. The results for LT, CD, and LBN with the detrended IRI data are reported in **Table 1** (the lower of the two rows of numbers associated with each participant). Inspection of **Table 1** reveals that Prediction 1 held for the data with and without the exponential cumulative growth.

We found strong support for Prediction 2. Paralleling notions in Refs. [11–13] we calculated search efficiency as $\eta = N/T$ where N is the total number of successful recalls and T is the time (in seconds) up to the last retrieval. For the data with exponential growth, the η values of the eight participants correlated positively with μ, $r = .96$ ($p < .001$), .95 ($p < .001$) and .92 ($p < .001$), for the LT, CD, and LBN estimates, respectively. For the detrended data the correlations were .87 ($p < .01$) for LT, .91 ($p < .01$) for CD, and .81 ($p < .01$) for LBN. As seen in **Table 1**, the μ values of highest η tended to ≈ 2, in agreement with the optimal search value expected from theory and numerical simulations [11–13].

In respect to Prediction 3, experimental data with and without detrending were compared to a simulated Lévy distribution using the model outlined in Refs. [6,11]. Inspection of **Fig. 3** suggests a fit between the data and the simulated Lévy distribution that was improved by detrending. In **Fig. 3b**, longer flights in the data with exponential growth tend to congregate toward the end of the series; in **Fig. 3c**, longer flights in the detrended data tend to be more evenly distributed, as in the simulated data of **Fig. 3a**. This latter observation, together with the relatively minor changes to μ when the exponential trend is removed (**Table 1**), suggests that the observed Lévy distribution of intervals between successive memory retrievals was not due to the increasing scarcity of targets and consequent decline in rate of cumulative recall.

Our results are compatible with the suggestion that memory is an adaptation to the statistical structure of the environment [17] and the proposition that statistical physics is well suited to the study of such phenomena [18]. The scale invariance of retrieval from semantic memory found in the present experiment mirrors the scale invariance of foraging and suggests that semantic retrieval, like foraging, is an adaptation to a domain (an environment) with fractal properties [19,20]. Relative to a random walk, the adaptive benefits of Lévy foraging/retrieving are fewer repeat visits [20] and higher search efficiency [11–13].
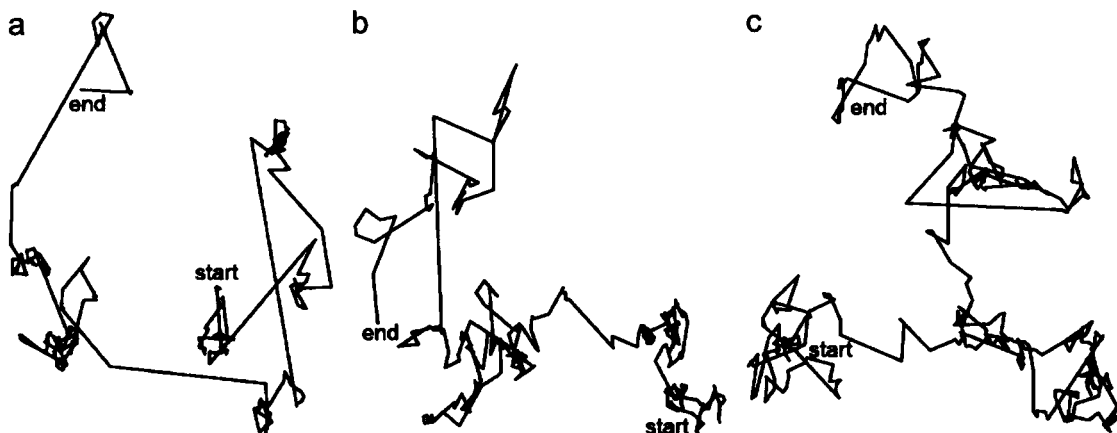

Fig. 3. Comparison of two-dimensional flights constructed from (a) a simulated Lévy distribution, (b) experimental data prior to detrending, and (c) experimental data after detrending, using the model outlined in Refs. [6,11]. The data for both (b) and (c) were of Participant 2 (most retrievals). The steps in (a) equaled those in (b) and (c). Clusters of similar length intervals interspersed with much longer intervals characterize the simulated and experimental distributions. A key difference is that the longer intervals are more evenly distributed in (a) and (c) than in (b) where they tend to cluster at the end of the flight. The similarity of (c) to (a) suggests that the Lévy dynamics of the basic semantic retrieval process may well approximate those of the nondestructive foraging model.

One implication of these latter remarks is that evolution's solution to optimizing the finding of specific foods might be evolution's solution to optimizing the finding of specific memories. Another implication is that landscape ecology [21] might prove more informative about memory's structure than formal systems of data organization.

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