DYNAMICAL SIMULATIONS OF A PHASE WINDOW MODEL OF RELATIVE TIMING

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ABSTRACT

We explore the hypothesis that intergestural phasing relations are implemented via coupling terms in a nonlinear dynamical systems model. We extend the dynamical model of relative phasing to include the possibility of attractor states that are characterized by phase windows, and contrast this behavior with standard models that display punctate relative phasing.

1. INTRODUCTION

Articulatory movements have been conceived of as coordinated, goal-directed gestures that are dynamically defined. Specifically, they have been modeled as critically damped oscillators that act as point attractors. An utterance is described not only by the parameter values of the component gestures but by how the gestures are coordinated or phased with one other. This approach captures coproduction by allowing gestures to overlap in time, with the acoustic consequences of the coproduced units reflecting their combined influence on the vocal tract.

Formal implementations of relative phasing among articulatory gestures have to date been in terms of invariant point-to-point phase relations such as constriction onset to target or onset to release. Such relations have been specified explicitly as part of an utterance's *gestural score* [1]. However, it has been empirically demonstrated that the observed relative phasing among articulatory gestures is not punctate. That is, the target phasing values are not single invariant points such as 180°, 229°, 240°, but rather are affected by linguistic factors such as prosody, stress, syllabification, etc. It has been hypothesized by Byrd [2] that the target values are constrained to fall within an admissible range or *phase window*.

Here we explore the hypothesis that intergestural phasing relations are implemented via coupling terms in a nonlinear dynamical systems model. In a set of simulations, we contrast the behavior of a standard model of coupled oscillators (limit-cycles) that displays punctate relative phasing with an extended model of relative phasing that displays stable phase windows. For example, if the system's phase window is defined from 220° to 245°, its steady state relative phase values can fall anywhere within this window. If the system's initial conditions are such that the relative phase is within the window, the system will remain in the window during its relaxation to a steady state. However, if the initial relative phase falls outside the window, the system's will settle to a steady state somewhere within the window. The particular value of steady state relative phase will depend on the initial conditions of the oscillators. Ultimately, it is hoped that this approach to relative timing can be incorporated into a dynamical model of intergestural articulatory control.

1.1. Motivating the Phase Window

Empirical studies of intergestural coordination in speech have repeatedly demonstrated systematic variability as a function of linguistic and extra-linguistic factors [3], such as syllable structure [e.g. 4, 5], stress, segment identity [6, 7], phrasal structure [e.g. 8], and speaking rate [e.g. 9]. The reasons for this principled variation may lie in the intrinsic communicative goals of speech for which articulatory movement patterns are governed in part by the perceptual capacities of listeners and the competing demands which the speaker-listener relationship necessitates. That is, temporal relations among phonological units are one means of communicating to the listener both the segmental identity of those units and their position in the hierarchy of linguistic syllabic and phrasal structure.

When a task dynamics for speech motor control was first proposed, Saltzman and Munhall [10] assumed that "gestures cohere in bundles corresponding, roughly, to traditional segmental descriptions, and that these segmental units maintain their integrity in fluent speech" (p. 365). They hypothesized that this cohesion is due to dynamical coupling of the gestures. Such cohesion is important since, for example, if the timing between the tongue dorsum gesture and the primary place gesture in a click is not strictly controlled, that segment will not be produced properly, namely with a velaric ingressive airstream. (See [2] for other examples of requisite intra-segmental timing.) No such dramatic constraints exist between gestures of separate segments or words, and evidence suggests that gestures forming a segment stand in a substantially tighter relationship than those which participate in different segments [11, 12].

In the present work we demonstrate that an appropriate type of coupling can yield a window of relative phasing between two coupled oscillators, lending further plausibility to the hypothesis that gestures belonging to single segments are coupled so as to yield narrow phase windows, while those belonging to different segments are coupled to yield wider phase windows [2]. By extension, cross-word timing may be even more variable, i.e., controlled by even wider phase windows.

1.2. Bridging the Gap

Nearly all quantitative accounts of temporal patterning in motor skills to date have focused on rhythmic (non-communicative) human actions. In particular, notions of relative phase have been formalized within a dynamical systems framework, primarily for concurrently performed rhythmic tasks [13, 14,]. Additionally, these formalisms have been limited to situations in which the attractor states of these coupled oscillatory systems display punctate steady state relative phasing among the component oscillators.

In this work we attempt to bridge the gap between accounts of relative timing in nonspeech rhythmic activities and relative timing in speech by demonstrating that relative phase windows can be instantiated in oscillatory systems as an alternative to punctate organization. It is our hope that this demonstration will ultimately provide insight into the dynamics of phasing among articulatory gestures, modeled as coupled second-order dynamical systems. In such a framework, phase windows would reflect both intrinsic and learned constraints on observed patterns of variability in the relative timing of gestural units [2].

2. MODELING RELATIVE PHASING

In this section, we present a method of flexibly controlling the relative phasing among coupled oscillators. This approach generalizes previous mathematical treatments in the motor control literature that use nonlinear dynamical systems theory. In our approach, the relative phase, ψ, between two oscillators is defined as an abstract *task-space* variable (or *order parameter*), and the ongoing motion of each component oscillator is defined in terms of a more physically-based set of *model articulator* variables. Relative phase, the state variable at the task space level, is defined as:

(1) $\psi = \phi_2 - \phi_1$, where

ϕ_i is the phase angle of the i[th] oscillator. The articulator variables for each oscillator are simply the respective sets of (position x, & velocity $\dot{x}$) state variables at the component oscillator level. We adopt a so-called *hybrid* oscillatory dynamics [15] at the model articulator level, defined by 2[nd] order equations of motion (see also [14]):

(2) $\ddot{x}_i = -\alpha_i \dot{x}_i - \beta_i x_i^2 \dot{x}_i - \gamma_i \dot{x}_i^3 - \omega_{0i}^2 x_i$, where

i denotes oscillator identity (i = 1, 2), α_i = linear damping, β_i = nonlinear van der Pol damping, γ_i = nonlinear Rayleigh damping, and ω_{0i} = linear natural frequency. These equations define independent, *uncoupled* motions for each oscillator.

In order for the oscillators to exhibit coordinated behavior with a desired pattern of relative phasing, the model articulator equations must be coupled to one another in a manner specific to the desired phasing. We do this, roughly, by fashioning a corresponding dynamics in the relative-phase task-space and transforming these dynamics into an appropriate set of coupling influences at the model articulator level. We begin by describing our approach in the context of producing punctate patterns of relative phase, in which the coupled system settles to particular relative phase values specified as single points in task space. We then show how our approach readily generalizes to systems that settle to ranges of relative phase values specified as windows in the task space.

Punctate relative phasing patterns can be produced by defining potential functions, $V(\psi)$, in the task space with single minima corresponding to the desired relative phase value, ψ_0. One of the simplest such potential functions (Figure 1a) is:

(3) $V(\psi) = -a\cos(\psi - \psi_0)$

where the height of the potential valley is equal to *2a*. This potential function can be used to define a 1[st] order equation of motion in task space according to the following general expression:

(4) $\dot{\psi}_T = -dV/d\psi = -a\sin(\psi - \psi_0)$

where $\dot{\psi}_T$ is the task-specific relative phase velocity (i.e., the system's desired task-level state velocity) that is a function of the current relative phase (i.e., task-space state). The resulting motion can be visualized as that of a ball rolling down the sides of the (sticky) walls of the potential surface until the ball comes to rest at the valley's minimum, ψ_0. We then convert this task-space state velocity into a corresponding set of model articulator coupling terms. This is done in several steps.

The first step is to transform each oscillator's current model articulator state, defined in terms of Cartesian position (x_i) and velocity ($\dot{x}_i$) coordinates on their respective phase planes, into a corresponding set of polar phase angle (ϕ_i) and radial amplitude (A_i) coordinates according to:

(5a) $\phi_i = -\tan^{-1}\left(\dfrac{\dot{x}_i / \omega_{0i}}{x_i}\right)$

(5b) $A_i = \sqrt{x_i^2 + (\dot{x}_i / \omega_{0i})^2}$

The articulator phase angles are used to define the current value for relative phase in the task space using Eq. 1. The desired relative phase velocity, $\dot{\psi}_T$, (i.e., task-specific "forcing") is then defined according to Eq. 4.

The next step is to transform each model articulator's equation of motion (Eq. 2) into a set of first order equations that define the current set of state velocities for oscillator phase and amplitude. We do this by differentiating Eq. 5 with respect to time in order to generate equations of motion that define each oscillator's polar state velocity ($\dot{\phi}, \dot{A}$) in terms of its Cartesian state velocity ($\dot{x}, \ddot{x}$).

(6a) $\dot{\phi}_i = \left(\dfrac{\partial \phi_i}{\partial x_i}\right)\dot{x}_i + \left(\dfrac{\partial \phi_i}{\partial \dot{x}_i}\right)\ddot{x}_i$, where

$$\frac{\partial \phi_i}{\partial x_i} = \frac{\dot{x}_i}{\omega_{0i} A_i^2} \text{ and } \frac{\partial \phi_i}{\partial \dot{x}_i} = -\frac{x_i}{\omega_{0i} A_i^2}$$

(6b) $\dot{A}_i = \left(\dfrac{\partial A_i}{\partial x_i}\right)\dot{x}_i + \left(\dfrac{\partial A_i}{\partial \dot{x}_i}\right)\ddot{x}_i$, where

$$\frac{\partial A_i}{\partial x_i} = \frac{x_i}{A_i} \text{ and } \frac{\partial A_i}{\partial \dot{x}_i} = \frac{\dot{x}_i}{\omega_{0i}^2 A_i}$$

In order to convert these expressions into functions of A_i and ϕ_i only, we specify $\ddot{x}_i$ according to Eq. 2 and substitute $x_i = A_i \cos\phi_i$ and $\dot{x}_i = -\omega_{0i} A_i \sin\phi_i$

We then transform the task-space state velocity, $\dot{\psi}_T$, defined by Eq. 4 into a corresponding set of phase velocities for each of the oscillators. This is done by differentiating Eq. 1 with respect time and defining the inverse of the resultant expression to get:

(7a) $\dot{\phi}_{1,T} = -.5\dot{\psi}_T$

(7b) $\dot{\phi}_{2,T} = +.5\dot{\psi}_T$, where

$\dot{\phi}_{i,T}$ is the task-specific contribution to oscillator-i's phase velocity that is added to the phase velocity component defined by Eq. 6a.

Next, the resultant set of polar state velocities are transformed into corresponding sets of Cartesian state velocities according to:

(8a) $$\dot{x}_i = \left(\frac{\partial x_i}{\partial A_i}\right)\dot{A}_i + \left(\frac{\partial x_i}{\partial \phi_i}\right)\dot{\phi}_i, \text{ where}$$

$$\left(\frac{\partial x_i}{\partial A_i}\right) = \cos\phi_i \text{ and } \left(\frac{\partial x_i}{\partial \phi_i}\right) = -A_i \sin\phi_i$$

(8b) $$\ddot{x}_i = \left(\frac{\partial \dot{x}_i}{\partial A_i}\right)\dot{A}_i + \left(\frac{\partial \dot{x}_i}{\partial \phi_i}\right)\dot{\phi}_i, \text{ where}$$

$$\left(\frac{\partial \dot{x}_i}{\partial A_i}\right) = -\omega_{0i}\sin\phi_i \text{ and } \left(\frac{\partial \dot{x}_i}{\partial \phi_i}\right) = -\omega_{0i}A\cos\phi_i$$

Finally, Eq. 8a, b is integrated numerically to generate position and velocity trajectories for each of the model articulator oscillators. After the system's initial transients subside, the oscillators settle into a stable coupled steady state with a constant (or average) relative phase equal to the desired punctate value, ψ_0.

In order to generalize the model to display stable phase *windows* in the steady state, we simply use the following task space potential function (cf., Eq. 3):

(9) $$V(\psi) = \frac{2}{1+e^{-\alpha z}}, \text{ where } \quad z = -(\cos[\psi - \psi_0] + 1)$$

This equation represents a cosine-shaped potential whose valleys have been flattened by the logistic "squashing function," commonly used in connectionist computational models, in which α defines the severity of squashing (Fig. 2; see [16] for a related use of squashing functions in a coupled oscillator framework). We chose this function for its relative simplicity. More elaborate functions would allow window width and flatness to be manipulated independently. As with Eq. 3, we use this potential function to define a 1st order equation of motion in task space:

(10) $$\dot{\psi}_T = -\,\mathrm{d}V/\mathrm{d}\psi = \frac{-2\alpha e^{-\alpha z}}{\left(1+e^{-\alpha z}\right)}\sin(\psi - \psi_0)$$

where $\dot{\psi}_T$ is the task-specific relative phase velocity (i.e., the system's desired task-level state velocity) that is a function of the current relative phase (i.e., task-space state).

3. SIMULATIONS

The following simulations demonstrate the effects of point vs. window potentials, initial relative phase, location of potential well centers (ψ_o), and intrinsic oscillator frequencies and damping coefficients. Initial conditions are specified as follows: the initial amplitude and phase of oscillator 1 are fixed at 2 and 0°; the initial amplitude of oscillator 2 is fixed at 2 and its initial phase is set according to the desired initial relative phase. In figures below showing the potential wells, '∘' marks the initial relative phase and '*' marks the final relative phase.

3.1. Relative phase potential shape and initial conditions

In the simulations reported in this section, ω_{0i}, β_i, and γ_i are equal to 1, and $\alpha_i = -1$, resulting in sinusoidal motions of each oscillator, that is, oscillations with a single frequency component evidenced by circular or elliptical orbits in the phase plane (e.g., Fig. 1, bottom). Except as noted, window or point centers of the potential well (ψ_o) are at 0°, initial relative phase is set at +140°, and simulation duration is 15 s.

3.1.1. Shape of the potential. When the centers of the point and window potential wells have the same value, here $\psi_o = 0°$, the resultant steady state relative phase is *not* the same. With the point potential (Fig. 1), the final relative phase is nearly 0° (ψ=+.0004°). However, with the window potential (Fig. 2), the oscillators settle to a relative phase inside the window but not at its center, ψ =+ 55°.

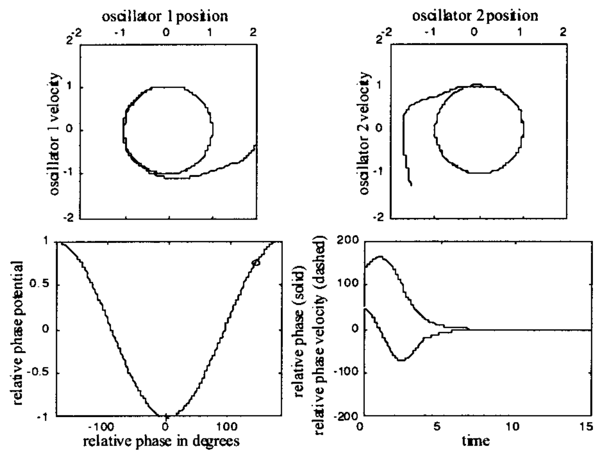

Figure 1. The point potential with sinusoidal oscillators.

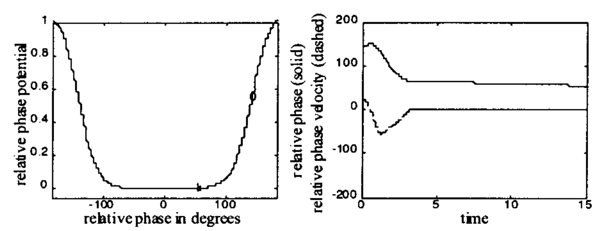

Figure 2. The window potential with sinusoidal oscillators.

3.1.2. Sensitivity to transient effects. Final relative phase is critically affected by the nature of initial transients in the case of the window potential but is not particularly affected with a point potential. Thus, when initial relative phase is altered from +140° to −140° with the window potential, final relative phase moves from 55° to −40°, but it is unaffected with the point potential.

3.1.3. Initial phase within window. When initial relative phase is already within the phase window, the final relative phase is only minimally affected, and may in fact move farther from the window center depending on the details of the initial transient behavior. Thus, $\psi_{initial}$=+20° yields ψ_{final}=+28° and $\psi_{initial}$=–20° yields ψ_{final}=–9°.

3.2. Relative phasing: Why are 0° and 180° special?

In contrast to the sinusoidal oscillations shown in Sec. 3.1, if the ω_{0i}, α_i, β_i, and γ_i are varied independently, the system will display nonsinusoidal, *relaxation* oscillations that have more than one frequency component. (All simulations in this section use $\omega_{0i} = 1$, $\alpha_i = -1$, $\beta_i = 5$, and $\gamma_i = 0$.) In this situation, the steady state behavior varies systematically as a function of the center value (ψ_o) of the point or window potential well. Fig. 3 shows a simulation of 30s duration using a point potential where ψ_o = 90° with an initial relative phase of –130° (= 90°+140° = +230° = 230°–360°). Note that relative phase does not settle to a constant value of 90°, but rather settles to an oscillatory steady state around 90°.

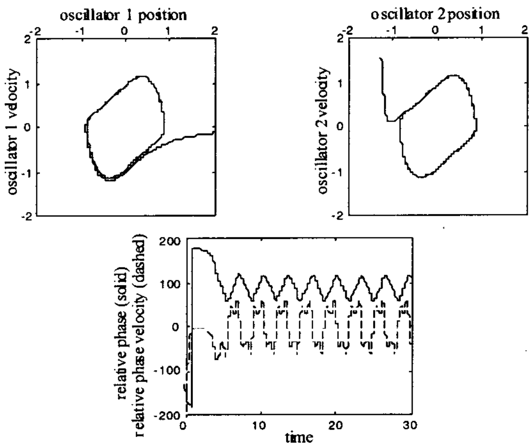

Figure 3. The point potential with relaxation oscillators.

This periodic behavior of relative phase is not observed in the sinusoidal oscillators because, in the latter case, the phase velocities of both oscillators are constant and equal to one another throughout their cycles. As a result, their relative phase easily settles to ψ_o with a relative phase velocity of 0°/s for *any* ψ_o after initial transient effects subside. When there are multiple frequency components, however, the phase velocity of both oscillators vary over their cycles. However, due to the *odd* symmetry of the limit cycle shapes (i.e., reflection through the phase plane origin; see Fig. 3), their relative phase velocity tends naturally to 0°/s when ψ_o = 0° or 180°. For all other values of ψ_o, relative phase velocity varies periodically around 0°/s. Figure 4 shows a simulation of 95s duration using a

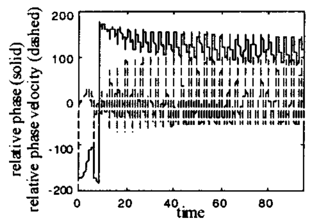

Figure 4. The window potential with relaxation oscillators.

window potential, with all other parameters the same as for Fig. 3. Note that relative phase settles to an oscillatory steady state about a value toward the window edge, not its center. Compared to behavior observed with the point potential, the mean value of the window potential's periodic relative phase trajectory takes much longer to settle into a steady state.

4. CONCLUDING REMARKS

We have demonstrated how relative phase between two coupled oscillators can be flexibly controlled within a nonlinear dynamical systems framework such that target relative phase, the state variable at the task space level, is defined as either a point or as a phase window. It is hoped that the phase window approach can be extended to the control of articulatory gestures, providing a mechanism for producing constrained variability in intergestural timing as a function of linguistic and paralinguistic factors.

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