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Role of the Inertia Tensor in Perceiving Object Orientation by Dynamic Touch

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Ss wielded an occluded L-shaped rod and attempted to perceive the direction in which the rod was pointing with respect to the hand. The pattern of the rod's different resistances to rotation in different directions, quantified by the inertia tensor, changes systematically with the rod's orientation. Perception of orientation by wielding is possible if the tissue deformation consequences of the rod's inertia tensor are detectable. It was shown that perceived orientation was a linear function of actual orientation for both free and restricted wielding and for rods of different-size branches. The eigenvectors of the inertia tensor were implicated as the basis for this haptic perceptual capability. Results were discussed in reference to information–perception specificity and its implications for effortful or dynamic touch.

It is not customary to think of the hand as an organ of perception given the dominant performatory aspect of everyday manipulations. The ordinary tasks of daily living require that objects be grasped, lifted, pushed, pulled, carried, inserted, turned, and so on. These tasks are usually executed under the guidance of the visual and haptic perceptual systems. The perceptual capabilities of the hand during manipulations, however, are often unnoticed because attention is directed at the performance capabilities, and awareness is dominated by what is seen rather than what is felt (Gibson, 1966). But deformations of skin and subcutaneous tissues, articulations of joints, and distortions of muscles and tendons are inevitable accompaniments of manipulation. It may well be the case that the haptic perceptual system plays a more fundamental role in the control of manipulatory activity than vision (Gibson, 1966).

When an occluded object is held firmly and wielded—that is, shaken, twisted, whipped back and forth, and so on—the perceptions one has of its dimensions are achieved through the haptic subsystem of effortful or dynamic touch (Gibson, 1966). The defining characteristic of this subsystem is that the information supporting perception comes from muscular effort. The extensions, compressions, and shearings of muscles and tendons underlie the perceptual capabilities of this subsystem more so than the deformations of skin and the articulations of joints. Experiments have shown that perceiving by dynamic touch is tied to an object's resistance to rotational acceleration. Moments of inertia, singly and in ratio, prove to be the relevant independent variables in perceiving the length of a rod or of a rod segment (Solomon & Turvey, 1988; Solomon, Turvey, & Burton, 1989a, 1989b) and in perceiving the shape of a regular object (e.g., cube, hemisphere, cone) (Burton, Turvey, & Solomon, 1991), respectively, by wielding. In the present article, we consider the role of rotational inertia in perceiving how a toollike object is oriented in the hand. Controlling acts involving hand-held tools and implements requires continuous perception of how the objects are disposed with respect to the organs of manipulation.

Moments and Products of Inertia and the Inertia Tensor

Consider Figure 1a, which depicts, for simplicity, the rotational motion of a thin, plane-rigid body around an arbitrary fixed axis labeled x with angular acceleration of $\dot{\omega}$ and angular velocity of ω. In the figure, the coordinate system Oxy has been located on the x axis with O as origin. One particle m of the body is shown with its tangential acceleration $\dot{\omega}y$ and its radial acceleration ω^2y. Directed against these accelerations are the inertia force $m\dot{\omega}y$ and the inertia or centrifugal force $m\omega^2y$ with moments about O of $m\dot{\omega}y^2$ and $m\omega^2xy$, respectively. The resultants of the very many moments of each type are obtained by integration: $\dot{\omega} \int my^2$ and $\omega^2 \int mxy$, respectively. The former integral is symbolized by I_{xx} and is referred to as the *moment of inertia* for rotation around x; the latter

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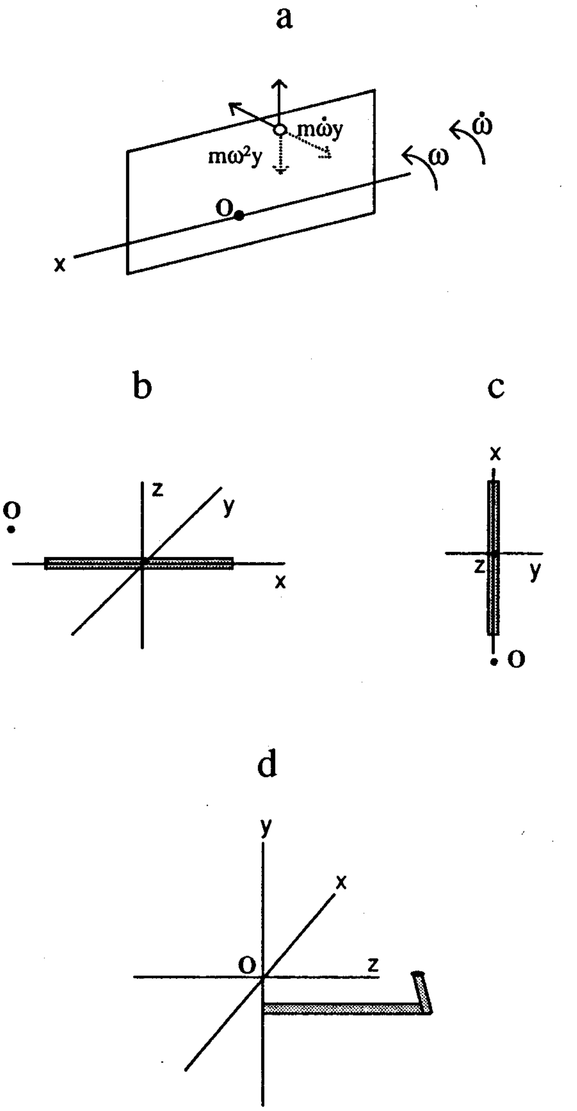

Figure 1. (a) The rotational motion of a thin, plane-rigid body about an arbitrary fixed axis with angular acceleration of $\dot{\omega}$ and angular velocity of ω (see text for details); (b) a side view and (c) a top view of a rod wielded about a fixed point O; (d) an L-shaped rod oriented at 45° with respect to a rectangular coordinate system with origin at O.

integral is symbolized by I_{xy} and is referred to as the *product of inertia* for rotation around x. For any real object rotating around any arbitrary axis, there are three moments of inertia (I_{xx}, I_{xy}, and I_{zz}) and six products of inertia (I_{xy}, I_{xz}, I_{yx}, I_{yz}, I_{zx}, and I_{zy}). Together they define the inertia tensor **I**, a way of quantifying an object's different resistances to rotation in different directions. **I** is represented by a 3 × 3 matrix with the moments on the diagonal and the products off the diagonal. It is a symmetric tensor in that products of inertia with the same subscripts ordered differently do not signify different quantities; for example, $I_{xy} = I_{yx}$.

It is always possible to find a set of axes through any point in an object for which **I** reduces to just moments of inertia (Goldstein, 1980). This maneuver is referred to as a *transformation to principal axes* (or eigenvectors) or as *diagonalizing the tensor*, and the moments of inertia are referred to as the *principal moments* (or eigenvalues). With regard to Figure 1a, one would find an x-axis such that for every centrifugal moment $m\omega^2xy$, there is a corresponding $m\omega^2x(-y)$ meaning that the resultant centrifugal moment obtained by integration would be equal to zero. Similarly, one could find an orthogonal y-axis such that each $m\omega^2yx$ would be canceled by a corresponding $m\omega^2y(-x)$. Clearly, the x- and y-axes in question are axes of reflectional symmetry: They would pass through the center of mass (CM). Because supports would be required to fix the axis in space coordinates (i.e., coordinates other than those anchored in the object), the importance of choosing axes that eliminate the products of inertia is that the supports do not have to provide additional forces (so-called dynamical or bearing forces over and above the static forces) to accommodate the object's rotation. The object is then said to be balanced.

One implication of the single-valued dependency of perceived rod length on rod moment of inertia found in previous research is that the haptic subsystem of dynamic touch can separate the moments of inertia from the products of inertia (Solomon & Turvey, 1988; Solomon et al., 1989a, 1989b). Appreciating this implication rests on a further understanding of the dependencies of moments and products on the point of rotation. When a rigid body is pivoted so that one point is fixed, it is convenient to choose that point to be the origin. If there is no fixed point, then the origin is generally chosen to be at CM. Consequently, it is useful to be able to relate the moments and products of inertia around an arbitrary origin (say, a point in the wrist) to those around the CM (say, of a hand-held rod). Relations of the following form are obtained: $I_{xx} = M(Y^2 + Z^2) + I^*_{xx}$, and $I_{xy} = -MXY + I^*_{xy}$ (Kibble, 1985). The first equation is the parallel axis theorem for moments of inertia, and the second equation is an equivalent theorem for products of inertia. Let us focus on the second equation. The term I^*_{xy} is the product of inertia that refers to CM as origin, X and Y refer to the distance of the arbitrary origin from the CM, and M is the mass. If the axes chosen at CM were the principal axes, then I^*_{xy} and all other products are 0. But as can be seen from the second equation, this does not mean that I_{xy} and the other products about the arbitrary axis are 0. In fact, this will be true only if the arbitrary point lies on one of the principal axes through the CM so that two of the three CM coordinates, X, Y, and Z, are 0.

In Figure 1b and 1c, we show a side and a top view, respectively, of a typical configuration in previous experiments (e.g., Solomon & Turvey, 1988) of the fixed point O and the rod. The fixed point in the wrist O has coordinates of (0, Y, $-Z$). Considering the sidewise dimensions of the rod to be negligible, the moments of inertia around O are, by the first equation above, $I_{xx} = I_{yy} = MD^2 + ML^2/12$ and $I_{zz} = 0$ (where D is the distance from O to CM); by the second equation above, the products of inertia around O are $I_{xy} = I_{xz} = 0$ and $I_{yz} = MYZ$. We can analyze, through multiple regression, a case like that depicted in Figure 1b and 1c, but with masses added at different positions on the rod to yield greater variety in the moments and products (e.g., Solomon et al., 1989a, Experiment 2). The analysis reveals that for each of 8 individual subjects, perceived rod extent was dependent on I_{xx} (= $MR^2 + MD^2/12$) only; there was no contribution of I_{yz} (= MYZ). As anticipated, one implication of the foregoing analysis is that in perceiving rod extent, dynamic touch separates the moments from the products—it achieves a kind of functional diagonalization of **I**. Another implication is that in perceiving rod extent, dynamic touch is sensitive to the diagonal form of **I**. This latter implication figures prominently in the present article.

Specificity of I to Object Orientation

We now proceed to the aspect of using tools and instruments that is the focus of the present investigation. When we hold and wield an object in the hand, we not only have an impression of its extent but also an impression of balance, of how the object is oriented with respect to the hand. This impression of orientation may be tied to **I**.

The off-diagonal terms of **I** reflect the asymmetry of an object (the uneven distribution of its mass) around its axis of rotation. As noted, they comprise centrifugal moments and are associated thereby with the reactive forces of the axes due to the rotation of the object. These latter reactions constitute dynamical forces in addition to the static reactive forces due to the weight of the object.

Consider an L-shaped rod (Figure 1d). The central principal axes of such an object are orthogonal Cartesian axes passing through its center of mass around which the products of inertia (and the dynamical reactions) are zero. If an L-shaped rod is grasped at one end and wielded by motions around the wrist, then by definition the axes of rotation will not be the central principal axes, products of inertia will exist, and dynamical reactive forces will be present throughout the wielding. The products of inertia for rotation around each of the axes in the wrist are constants; they comprise invariants over the activity of wielding. Now it is important to note that for a fixed system of coordinates with origin at the wrist, the products of inertia and the moments of inertia will vary with how the rod is placed in the hand. Let the longer piece of the L-shaped rod be called the *stem*, let the shorter piece be called the *branch*, and let the stem be the piece that is held. Then it is clear that the rod can be placed in the hand such that if the longitudinal axis of the stem parallel to the body's sagittal plane is the x-axis, then the branch can be oriented in the yz plane at any angle between 0° and 360°. For each orientation of the rod—meaning the angle made by the branch in the yz plane—there will be a different alignment of the rod's mass in the wrist-based coordinate system.

In Table 1, we present the orientation-dependent **I**s for an L-shaped wooden rod, comprising a stem of 35 cm and 28 gm and a branch of 9 cm and 7.2 gm, held firmly in the hand at eight different orientations (Θ) corresponding to 0°, 45°, 90°, 135°, 180°, 225°, 270°, and 315°. The angles are the orientations of the branch of the L-shaped rod in the coordinate system defined through a fixed point in the wrist and depicted in Figure 1d. The tensors are calculated for a system of rectangular coordinates (see Figure 1d) with origin at a point O in the wrist (taken to be a mean perpendicular distance from the stem's longitudinal axis of 7 cm). The x- and z-axes are parallel to the ground plane, and the y-axis is perpendicular to the ground plane. (The calculations use repeated applications of the parallel axis theorems for moments and products, and Mohr's theorem [den Hartog, 1950; Kibble, 1985].) Inspection of Table 1 reveals how the components of **I** are defined uniquely for a given angle of orientation and how they change with angle. It is apparent from inspection that whereas **I** at each angle is specific to the angle, there is no single component of **I** that is angle specific. In the experiments of the present article, we entertain the hypothesis that if the muscular and tendon deformation consequences of wielding an L-shaped rod are specific to **I** (or some aspect of **I**) and are detectable, then the direction in which the branch of an L-shaped rod points is perceptible by dynamic touch.

Experiment 1

The goal of this experiment was to ascertain whether a person could determine (without the aid of vision) the orientation in the hand of the L-shaped rod giving rise to the orientation-specific **I**s reported in Table 1. Because of the lightness of the materials and the nearly fourfold difference between stem and branch, the rod had no tendency to rotate under its own weight when held in the hand at any angle. The rod had to be wielded to determine its orientation.

Method

Subjects. Six persons (3 men and 3 women) associated with the University of Connecticut participated as volunteers. All subjects were right-handed.

Table 1
Inertia Tensors of the L-Shaped Rod Used in Experiment 1 as a Function of Θ

	Tensor components (g·cm²)					
Θ (deg)	I_{xx}	I_{yy}	I_{zz}	I_{xy}	I_{xz}	I_{yz}
0	22,524.13	20,562.14	1,967.32	0	0	−4,970.91
45	22,545.05	20,586.12	2,098.91	−104.09	706.86	−5,239.73
90	22,814.94	20.719.61	2,416.60	−224.64	999.65	−5,970.56
135	23,211.36	20,640.87	2,734.29	−213.60	706.86	−6,701.40
180	23,421.69	20,562.14	2,865.88	0	0	−6,970.21
225	23,211.36	20,640.87	2,734.29	213.60	−706.86	−6,701,40
270	22,814.94	20.719.61	2,416.60	224.64	−999.65	−5,970.56
315	22,545.05	20,586.12	2,098.91	104.09	−706.86	−5,239.73

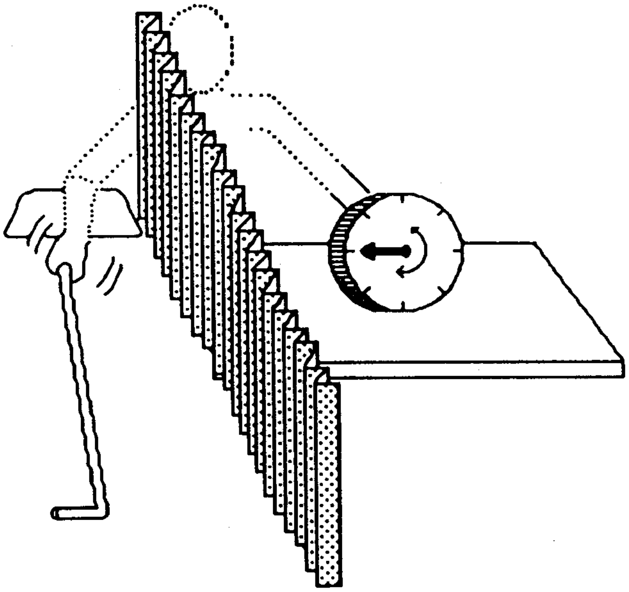

Figure 2. The experimental arrangement.

Materials. The L-shaped rod was made of two cylindrical pieces of pine. One piece (the stem) measured 35.0 cm, and the other (the branch) measured 9.0 cm. The radius of the wood was 0.64 cm, and the density was 0.59 g/cm^3, such that the mass of the total apparatus was 32.7 g. The branch was attached to the stem at a right angle. Electrical tape was wound around the first 6.0 cm of the stem so that subjects could hold the rod with minimal slipping. A smaller rod of the same relative proportions (stem, 23.0 cm; branch, 7.5 cm) was used as a sample.

Apparatus. The experimental arrangement is depicted in Figure 2. The subject sat with his or her right forearm resting on a horizontal writing surface attached to the seat. A screen separated the resting surface from a small desk that supported the report board. The report board was made of cardboard and had a cardboard disk facing the subject, with no markings other than an arrow and a handle on the edge of the disk above the arrow. On the side of the board facing the experimenter was a disk made of two plastic protractors that was connected to the paper disk. Thus, as a subject moved the arrow on his or her side, the experimenter could read the angle at which the subject had set the arrow.

Procedure. The subject's task was to shake the unseen L-shaped rod in his or her right hand and report the angle at which the branch was pointing. At the branch end of the stem, there were eight equidistant markings corresponding to eight angles at 45° intervals. To position the L-shaped rod so that the branch was at 270°, for example, the experimenter merely had to turn the rod so that the marking of "270" faced upward. Of course, these markings were reversed from the experimenter's perspective to be accurate to the facing subject.

The rod was positioned in the subject's hand, with the thumb on top of and parallel to the handle, and the subject was instructed to return his or her hand to this configuration before making the judgment. The subject was allowed to move the hand in relation to the wrist but was not permitted to move the L-shaped rod within the hand; that is, the handle could not be rolled within the hand. During each trial, the subject's wielding was closely monitored. Any trial in which the handle was seen to move in relation to the hand, or the wrist was seen to move in relation to the arm rest, was repeated. Each subject was shown the sample L-shaped rod before starting; no subject ever saw the actual L-shaped rod.

No feedback of any sort was provided to the subject throughout the course of the experiment. Eight settings of the rod apparatus were used, ranging from 0° to 315° in 45° steps, with the branch pointing straight upward for a setting of 0°. The subject was not informed that the number of possible positions of the rod were limited in this way. There were four repetitions of each orientation. The rod could be wielded in any way preferred as long as the wrist stayed at the designated place on the arm of the chair and the stem stayed in the same place in the hand. There was no time limit within a trial for shaking the L-shaped rod. Each person reported his or her judgment by turning the circular dial mounted on the visible report board until the arrow on the dial corresponded with the perception of the orientation of the occluded L-shaped rod.

Results and Discussion

Table 2 shows the pattern of subjects' responses for each of the target orientations. For each target orientation, there were 24 responses (6 subjects × 4 repetitions of each orientation). Responses are grouped into orientation sectors of 45° magnitude ranging from 22.5° below each target position to 22.5° above each target position; for example, the 0° response sector extends from 338.5° to 22.5°; the 45° response sector extends from 23.5° to 67.5°, and so on. Inspection of Table 2 reveals that given a target orientation, most responses fell into the three sectors comprising the target sector and the two sectors most immediately adjacent to it (71% for 0°, 54% for 45°, 96% for 90°, 83% for 135°, 58% for 180°, 67% for 225°, 67% for 270°, and 67% for 315°).

Table 2
Proportion of Responses Falling Within 45° Sectors Centered at Each Target Θ in Experiment 1

	Sector of perceived orientation (deg)							
Θ (deg)	0	45	90	135	180	225	270	315
0	.46	.125	0	.17	.04	0	.08	.125
45	.21	.21	.125	.21	0	0	.04	.21
90	0	.17	.54	.25	.04	0	0	.04
135	.04	.125	.25	.50	.08	0	0	0
180	.08	.04	.21	.25	.33	0	.08	0
225	.08	.04	0	.17	.38	.25	.04	.04
270	.08	.04	0	.08	.125	.29	.25	.125
315	.21	.08	.08	.04	.04	.08	.29	.17

Circular statistics. An arithmetic mean is not suitable for judgments of angle because of the periodic nature of a circle. For example, for a rod at 315°, a judgment of 15° and a judgment of 255° are both within 60° and should be considered equally accurate. The arithmetic average of these two judgments would be 135°, however, which differs from the setting by 120°. Trigonometric functions are suitably circular, but they cycle twice in a circle, and thus there would not be a unique value for every point on the circle. The preceding difficulties are overcome by circular statistics (Batschelet, 1965, 1978). The main component of this method is finding an average judgment by taking the sine and cosine of each judgment, summing these judgments over repetitions, and transforming this sine and cosine combination back into the angle they pertain to (referred to as α). This method yields the resultant of the vectors described by each of the component angles.

There were four circular statistics of interest: α, the measure of the resultant angle of the judgments; r, a measure analogous to a correlation coefficient, which measures the coherency of the various judgments (technically, the length of α in a unit circle); Rayleigh's z, which is analogous to an F ratio and assesses the significance of the r value; h, the homing coefficient, which measures how closely the perceived orientations "homed in" on the actual target orientation. The h is like the r coefficient, except that it takes into account whether the judgments were accurate (technically, it is the length of the vector in the unit circle that points to the home direction). Thus, if most subjects judged the rod set at 45° as 180°, they might have a high r but a low h.

Table 3 shows the principal circular statistics for three examples in which the correct setting is 105°. In Group 1, consider the sum of cosines to be the value on the y-axis of a Cartesian coordinate system and the sum of sines to serve as the x value (see Figure 3). α is the vector from the origin to this point, that is, the resultant of the y vector and the x vector. The length of this vector is derived straightforwardly from the Pythagorean theorem and equals the square root of $(\Sigma\cos)^2 + (\Sigma\sin)^2$, which in the case of the values from Table 3 is equal to 6.95. This value is symbolized by a capital R, and when divided by n it yields r, or .99 in this example. Rayleigh's z, in turn, is equal to n times the square of r, or 6.92 in the first example.

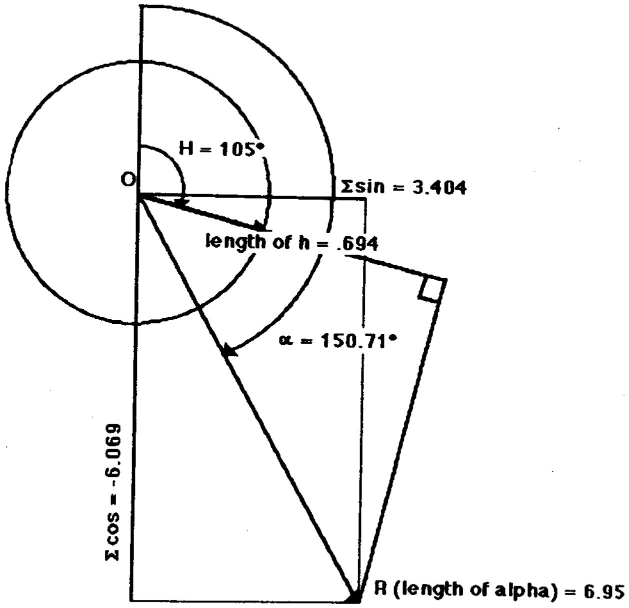

Figure 3. The relationship of α, Θ, R, and h under Batschelet's circular statistics.

Table 3
Principal Circular Statistics for Three Examples, Given a Correct Setting of 105°

Statistic	Group 1	Group 2	Group 3
Judgments (deg)	145	105	105
	150	140	100
	155	70	95
	140	180	110
	150	30	90
	155	120	105
	160	90	100
Σcos	−6.069	−0.393	−1.294
Σsin	3.404	4.914	6.837
α (deg)	150.71	94.57	100.71
r	.994	.704	.994
Rayleigh's z	6.917	3.472	6.916
h	.694	.692	.991

Now, consider a vector drawn from the origin to the home direction and another vector drawn from the end of α and intersecting the home vector at a right angle (see Figure 3). The distance between this intersection and the origin on a unit circle is h. It can be calculated by taking the cosine of the angular difference between α and the home vector and multiplying this cosine by r. In this example, it is equal to .69.

We now compare the three groups of scores in Table 3. For Group 1, the correct setting is 105°, but the judgments cohere closely to 151° (as revealed by the α value). Thus, r and Rayleigh's z are high, but h is lower because the judgments were somewhat far off track. For Group 2, judgments are much closer to 105°, but they are not as coherent. Consequently, the h is basically the same as in Group 1 but with the r and Rayleigh's z lower than in Group 1. Group 3 represents an ideal case: The data are coherent and accurate, and r, h, and z are all high. The comparison of groups makes clear how h takes into account both the dispersion of judgments about α and how far α deviates from Θ. An h of 1.0 would be perfect homing.

Dependency of α on Θ. Returning to the actual data of the present experiment, Figure 4 shows the regression of the α values on Θ for each person and for the group. Individual and group data are fit by linear functions, demonstrating that perceived orientation α followed actual orientation Θ. The potential confusability of certain directions was in evidence, however. As can be seen from Figure 4, 2 subjects (Subjects

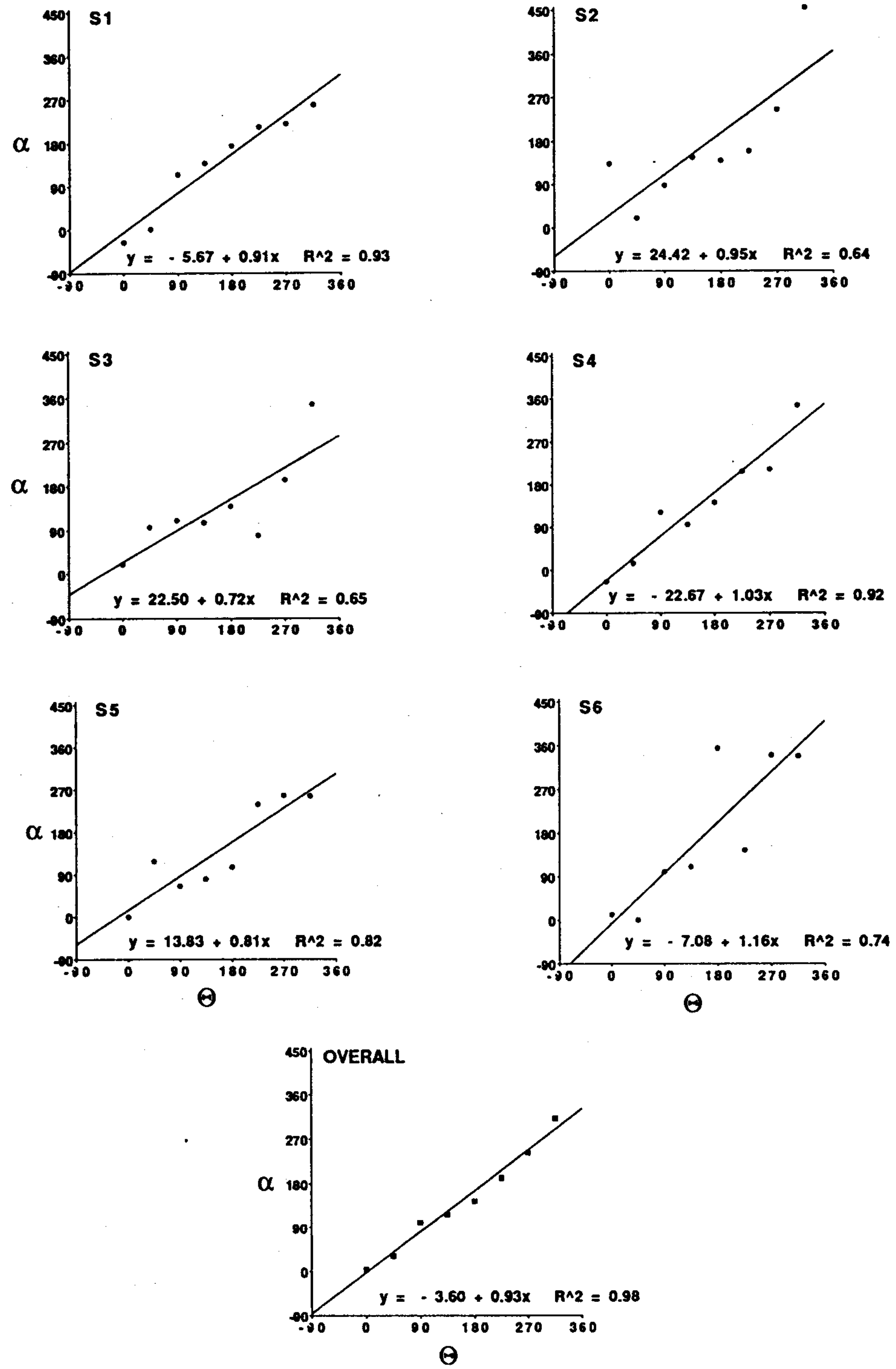

Figure 4. The regression of α on Θ for each subject and for the average data in Experiment 1. (For $\Theta = 0°$ and $\Theta = 315°$, points are plotted at the appropriate point of a circle closest to the setting [e.g., a response of 45° to a setting of 315° would be plotted as 405°, and a response of 315° to a setting of 45° would be plotted as −45°].)

2 and 6) reported essentially the opposite orientation from the one presented: 135° for 0° (Subject 2) and 340° for 180° (Subject 6). Inspection of Table 1 suggests that if perceived orientation was affected in particular by I_{xy} and I_{xz}, then confusion between 180° and 0° (and, similarly, between 360° and 180°) would be expected.

r, Rayleigh's z, and h. Table 4 shows the means of the principal circular statistics for the eight actual orientations of

Table 4
Mean Circular Statistics as a Function of Θ in Experiment 1

	Statistic			
Θ (deg)	α (deg)	r	z	h
0	4.06	.54	6.61	.52
45	31.45	.49	5.67	.47
90	100.04	.86	17.64	.84
135	116.24	.79	15.10	.75
180	143.98	.57	7.70	.46
225	191.36*	.51	6.33	.43
270	242.04	.51	6.18	.45
315	311.51	.44	4.62	.44

* $p < .05$.

Experiment 1. From the r measures, it can be inferred that there was reasonable coherency among the responses of the 6 subjects at each rod orientation. For the 90° orientation, the coherence of judgments across trials and subjects was especially high. The Rayleigh's z proved to be significant, $z(24) = 2.96$, $p = .05$, for each of the eight settings, confirming the inference. Turning to the h values, which index the tendency for the judgments to cohere around the target orientation, it seems that the different orientations may have differed in the accuracy with which they were perceived; the h values were much higher for 90° and 135° (as would be expected from inspection of Table 2). A method for deciding on the proximity of a given α value to the actual orientation was given by Stephens (1962). The Stephens test finds a critical value for R derived by the previously mentioned methods. This critical value, called R_0, is found (using the approximate tests) by applying one of five formulas to the h statistic, with the particular formula to be used dependent on how close h is to r. (Note that Stephens does not always use the same symbols as Batschelet, but they have been "translated" here for simplicity.) A significant result indicates that the α value found is significantly different from the home direction. Because we wished to assess if α was significantly close to Θ, in other words, if α was significantly accurate, it was necessary to reverse the Stephens test by subtracting the actual r value from 1 before proceeding with the test. As can be seen from Table 4, α at 225° differed significantly from 225°, all other αs were statistically identical with their Θs.

Evaluating I. To evaluate the dependency of perceived orientation on **I**, the 48 α values (8 for each of 6 subjects) were regressed on I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, and I_{yz} as given in Table 1. The multiple regression yielded $r^2 = .74$ ($p < .0001$) with the following significant terms: I_{xx}, $p < .01$; I_{zz}, $p < .0001$; I_{xy}, $p < .0001$; I_{xz}, $p < .0001$; I_{yz}, $p < .001$. The partial F values of these terms were, respectively, 10.35, 18.43, 21.5, 40.68, and 15.36, suggesting that off-diagonal terms contributed more than the diagonal terms. Because **I** is being advanced as the proper variable rather than Θ (**I** can affect the body's tissues, Θ cannot) it must at least account for the same amount of variance as Θ. The simple regression of the 48 α values on Θ yielded $r^2 = .74$ ($p < .0001$). The results of the experiment are consistent, therefore, with the hypothesis that if the muscular and tendon deformation consequences of wielding an L-shaped rod are specific to **I** (or some aspect of **I**) and are detectable, then the direction in which the branch of an L-shaped rod is pointing is perceptible by dynamic touch.

Experiment 2

In making their judgments in Experiment 1, subjects engaged in highly varied wielding maneuvers to determine the orientation of the hand-held rod. Although the position of the bony structure of the elbow on the arm of the chair was fixed, the forearm could be rotated around its longitudinal axis. (By just rotating the forearm, the right hand could assume any position clockwise between approximately 270° and 180° and no positions clockwise between approximately 180° and 270°.) Moreover, the hand could be moved in the two orthogonal planes through the longitudinal axis of the forearm, and it could be circumducted. For illustration, consider the movements permitted for the single posture of the forearm in which the thumb, lying along the main axis of the rod stem, points directly forward with thumbnail up. In this posture, the rod could be wielded in (a) a plane parallel to the sagittal plane at many angles to the transverse plane, (b) a plane parallel to the transverse plane at many angles to the sagittal plane, (c) a plane parallel to the frontal or lateral plane, or (d) cones of many different solid angles. In this experiment, we limit subjects to (a) or (b) on some trials and let them explore freely; that is, we allow (a)–(d) on other trials. The questions of interest are whether h and the mapping of α to Θ depend on the conditions of exploration?

Method

Subjects. Seven (5 women and 2 men) undergraduates at the University of Connecticut participated as partial fulfillment of course requirements. Six of the subjects were right-handed.

Materials. The same L-shaped object was used as in Experiment 1.

Apparatus. The same room arrangement and report apparatus were used as in Experiment 1.

Procedure. The Θ values were the eight used in Experiment 1. There were three types of trials defined by three styles of exploration, namely, shaking the rod in any way desired (free trials); shaking it from side to side in a plane parallel to the transverse plane (horizontal trials); and shaking it up and down in a plane parallel to the sagittal plane (vertical trials). These three conditions were intermixed randomly, and the subject was informed before each trial what style of shaking was required. These constraints were adhered to strictly. If a subject accidentally shook the L-shaped rod inappropriately, that particular trial was repeated later. No subject required more than two of these repetitions. As in the preceding experiment, subjects were told nothing else about the setting of the rod for that trial. There were three repetitions of eight settings with three styles plus six practice trials (including trials of all three styles), giving 78 trials.

Results and Discussion

The α dependence on Θ as a function of style of wielding. Figure 5 shows for each of the three wielding conditions the 56 computed α magnitudes (8 for each of 7 subjects) plotted against Θ. For each condition, the dependency of α on Θ was linear. To determine whether the three linear

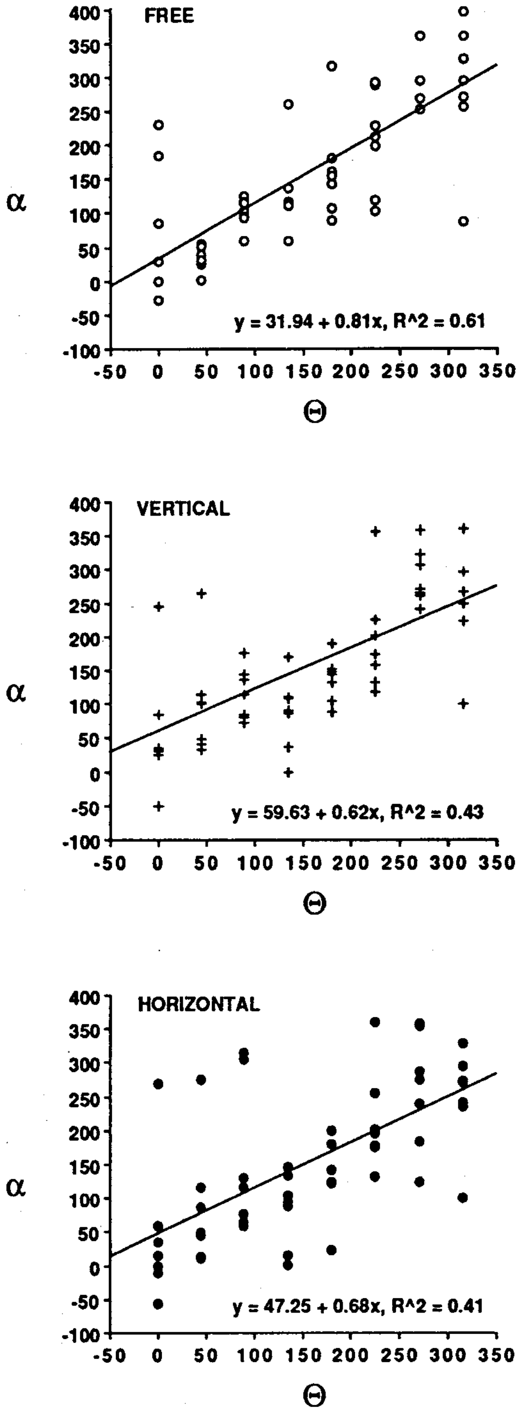

Figure 5. Regressions of α against Θ for each of the three wielding conditions of Experiment 2. (The data of all 7 subjects are included in each panel.)

functions differed, we conducted a multiple regression with the continuous independent variable of Θ, the categorical independent variable of exploration or wielding condition, and subjects. The categorical variable was coded as two vectors with the interaction terms given by the products Θ (Vector 1) and Θ (Vector 2); six vectors were used to code the 7 subjects (Pedhazur, 1982). The multiple regression, using all 168 α values, yielded $r^2 = .51$. Neither the interactions nor the vectors proved significant ($F < 1$ in both cases), meaning that the linear functions shown in Figure 5 do not distinguish between either slope or intercept, respectively. Repeating the multiple regression analysis at the level of the individual subjects revealed that for 2 subjects the resultant functions did distinguish between both intercept and slope. The 2 subjects, however, did not exhibit similar patterns. For 1 subject, the slope of free wielding differed from the vertical slope but not the horizontal slope; for the other subject, the slope of free wielding differed from the horizontal slope but not the vertical slope.

The h dependence on style of exploration. The regression equations reported in Figure 5 reveal that the variance in α due to Θ was largest for the freewielding condition. Table 5 shows the principal circular statistics for the three styles of wielding. We computed each statistic per orientation from 21 observations. Inspection of Table 5 suggests that higher levels of accuracy, as measured by h, were achieved under the free wielding conditions. An analysis of variance (ANOVA) on the h values found a significant effect of wielding style, $F(2, 14) = 3.87$, $p < .05$. A Tukey test found that free and horizontal were different to the .05 level, whereas free and vertical, and horizontal and vertical, were not significantly different.

Table 5
Mean Circular Statistics as a Function of Styles of Wielding and Θ in Experiment 2

		Statistic			
	Θ (deg)	α (deg)	r	z	h
Free wielding					
	0	3.65	.39	3.19	.39
	45	47.92	.81	13.69	.81
	90	100.89	.81	13.64	.79
	135	112.87	.61	7.69	.57
	180	143.68	.59	7.32	.48
	225	210.43	.35	2.64	.34
	270	271.42	.60	7.52	.60
	315	305.94	.31	2.04	.31
Vertical wielding					
	0	2.19	.44	4.13	.44
	45	48.22	.43	3.91	.43
	90	75.54	.40	3.41	.39
	135	94.62*	.49	5.00	.37
	180	154.10	.55	6.37	.50
	225	196.53	.54	6.08	.47
	270	257.99	.26	1.38	.25
	315	267.71*	.51	5.45	.35
Horizontal wielding					
	0	−11.10	.25	1.29	.24
	45	72.42	.52	5.59	.46
	90	108.26	.69	9.85	.65
	135	82.27*	.56	6.57	.35
	180	138.31*	.51	5.48	.38
	225	180.46**	.20	0.81	.14
	270	296.33	.58	6.95	.52
	315	280.62*	.40	3.40	.17

* $p < .05$. ** $p < .01$.

With respect to the match of α to Θ, Table 5 shows that the match was statistically perfect for all orientations under free wielding, statistically imperfect for 135° and 315° under vertical wielding, and statistically imperfect for 135°, 180°, 225°, and 315° under horizontal wielding.

*The α dependence on **I**.* The 168 α values (24 for each of 7 subjects) were regressed on I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, and I_{yz} as given in Table 1. The multiple regression yielded $r^2 = .53$ ($p < .0001$) with the following significant terms: I_{xx}, $p < .0001$; I_{zz}, $p < .005$; I_{xy}, $p < .01$; I_{xz}, $p < .0001$; I_{yz}, $p < .02$. The partial F values of these terms were, respectively, 33.82, 8.26, 6.29, 25.98, and 5.13, suggesting an equitable contribution by both diagonal and off-diagonal terms. Simple regression revealed that Θ accounted for 48% of the variance in α; the hypothesis under study requires that **I** accounts for α at least as well as Θ accounts for α.

Experiment 2 was directed primarily at whether h and the mapping of α to Θ depended on the conditions of exploration. Taking all analyses into consideration, the answer appears to be that exploration style affected h but not the form of the dependency of α on Θ. The branch of the L-shaped rod tended to be perceived as pointing in the same direction for free, vertical, and horizontal wielding, but not with equal accuracy. Perceiving orientation seemed to be best when the person could explore freely the full range of three-space motions—and, therefore, the full range of muscular and tendon deformations—permitted by the conditions of the experiment. Nonetheless, it was apparent that reliable discrimination among orientations was achieved under restricted explorations. The information that specifies orientation can be obtained reasonably successfully under conditions of wielding restricted to a vertical plane and under conditions of wielding restricted to a horizontal plane, although it may be obtained more successfully in the former than in the latter case.

Experiment 3

To be useful, the means by which one perceives the orientation of an object in the hand should be indifferent to the size of the object: Objects of different sizes but at the same orientation should be perceived at the same orientation. It might be expected, however, that the accuracy of perception would depend on size. Larger versions of the same object will be characterized by larger rotational moments, suggesting that the asymmetry of the mass distribution in relation to the hand should be more easily detectable. In terms of circular statistics, α should be affected by Θ but not by size, and h should be affected by size and not by Θ.

In this experiment, three L-shaped rods distinguished by the size of the branch (6.0, 11.5, and 16.25 cm) were used. Table 6 gives the details of the **I**s for each of the three rods at each of the eight Θ values. (The tensor calculations are with respect to the same rectangular coordinate system used to calculate the **I**s in Table 1.) In addition to permitting an evaluation of the preceding two hypotheses, Experiment 3 provides a more analytic evaluation of the contribution of **I**. Given that very different patterns of resistances to rotation are associated with the same Θ, we can conduct a stronger test of whether **I** predicts α as well as Θ predicts α.

Table 6

Inertia Tensors of the Three L-shaped Rods Used in Experiment 3 as a Function of Θ

		Tensor components (g·cm²)					
	Θ (deg)	I_{xx}	I_{yy}	I_{zz}	I_{xy}	I_{xz}	I_{yz}
6-cm branch							
	0	12,949.45	11,333.66	1,621.38	0	0	−3,578.66
	45	12,980.54	11,352.95	1,671.76	−48.13	232.92	−3,668.53
	90	13,082.88	11,372.23	1,793.39	−86.00	329.39	−3,908.05
	135	13,223.79	11,352.95	1,915.02	−73.50	232.92	−4,147.56
	180	13,293.46	11,333.66	1,965.39	0	0	−4,237.44
	225	13,223.79	11,352.95	1,915.02	73.50	−232.92	−4,147.56
	270	13,082.88	11,372.23	1,793.40	86.00	−329.39	−3,908.05
	315	12,980.54	11,352.94	1,671.76	48.13	−232.92	−3,668.53
11.5-cm branch							
	0	16,880.59	15,254.66	1,632.39	0	0	−3,564.58
	45	16,928.02	15,416.32	1,841.46	−138.39	966.62	−3,917.30
	90	17,271.12	15,577.97	2,346.23	−356.92	1,367.00	−4,931.58
	135	17,937.54	15,416.32	2,850.99	−366.37	966.62	−5,945.87
	180	18,308.27	15,254.66	3,060.06	0	0	−6,298.59
	225	17,937.54	15,416.32	2,850.99	366.37	−966.62	−5,945.87
	270	17,271.12	15,577.97	2,346.22	356.92	−1,367.00	−4,931.59
	315	16,928.02	15,416.32	1,841.46	138.39	−966.62	−3,917.30
16.25-cm branch							
	0	20,372.15	18,640.98	1,738.37	0	0	−2,986.14
	45	20,324.20	19,121.68	2,171.11	−176.79	2,000.69	−3,679.74
	90	20,888.24	19,602.39	3,215.86	−738.75	2,829.41	−5,815.55
	135	22,413.69	19,121.68	4,260,61	−867.97	2,000.69	−7,951.35
	180	27,246.00	18,640.98	4,693.36	0	0	−8,644.96
	225	22,413.69	19,121.68	4,260.61	867.97	−2,000.69	−7,951.35
	270	20,888.24	19,602.39	3,215.86	738.75	−2,829.41	−5,815.55
	315	20,324.20	19,121.68	2,171.11	176.79	−2,000.69	−3,679.74

Method

Subjects. Eight persons (6 women and 2 men) associated with the University of Connecticut participated for a fee of $4. One subject declined the fee. All subjects were right-handed.

Materials. Three rods were constructed with the same design as in Experiment 1. Each had a stem of 30.0 cm; three different lengths were used for the branch, 6.0 cm, 11.5 cm, and 16.25 cm. The three L-shaped rods are referred to as the short, the medium, and the long; the medium rod is closest to the length used in Experiment 1. The same sample L-shaped rod was used as before. As remarked above, Table 6 gives the details of the orientation-dependent Is for each of the three rods.

Apparatus. The same room and equipment were used as in Experiment 1.

Procedure. The task was the same as in Experiment 1. Each participant was instructed that L-shaped rods of different lengths would be placed into the right hand. They were not told how many different rods there were, nor were they told anything about the rod in the right hand on any particular trial. The same eight orientations were used as in Experiment 1, but to reduce the number of trials per subject, each subject only encountered six of the orientations. For a given subject, the same six orientations were used for each branch length, with three trials at each orientation. There were 54 trials (18 trials for each branch). The experiment was designed so that across subjects each orientation was used an equal number of times (six), with the two orientations that were omitted from a given subject's trials drawn from different quadrants. As before, no subject was told the number of possible orientations.

Results and Discussion

For each subject, we computed the α and h values of the subject's responses for each of the six orientations experienced by the subject with each of the three L-shaped rods. Table 7 summarizes the individual linear regressions of α on actual orientation Θ. Only 3 of the 24 regressions proved insignificant: two for the short rod and one for the medium rod. As inspection of Table 7 reveals, the overwhelming tendency in the data was for α to increase at a rate commensurate with the increase in Θ. Experiment 3 therefore corroborates and extends Experiments 1 and 2 in demonstrating the ability of people to perceive, by wielding, the orientation of occluded hand-held objects. It also suggests that this ability (expressed through subjects lacking any explicit practice in the task) can be exhibited with near-perfect accuracy as branch size increases in relation to stem size. Inspection of Table 7 shows that for 6 of the individual subjects, variance in α was almost completely accounted for by Θ when the branches were 11.5 cm and 16.25 cm.

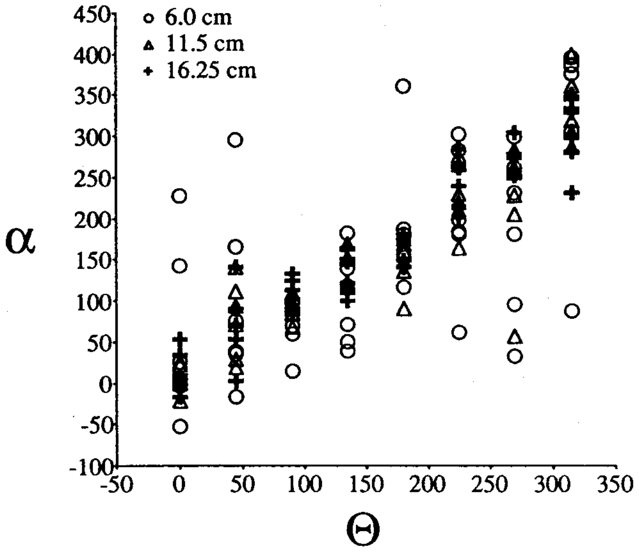

Figure 6. Dependence of α on Θ for each L-shaped rod in Experiment 3. (The data of all 8 subjects are included.)

Figure 6 presents for each L-shaped rod the linear regression of α on Θ for all 48 values (8 subjects × 6 angles). Clearly, performance with the 6-cm branch was different from that with the 11.5- and 16.25-cm branches. What precisely was the nature of this difference? Was it with respect to α or h?

Dependency of α and h on Θ and L. For each subject, the 18 α quantities and the 18 h quantities were multiply regressed on Θ and branch size, L. The results of the α regressions are shown in Table 8, and those of the h regressions are shown in Table 9. As can be seen, α was affected by Θ but not by L, whereas h was affected by L and not by Θ. An ANOVA on

Table 7

Results of the Linear Regression Analysis of α on Θ as a Function of the Size of the L-Shaped Rod for Each Subject in Experiment 3

	Branch size (cm)		
Subject	6	11.5	16.25
1			
Slope	1.47	1.10	0.89
Intercept	−57.60	−14.30	26.10
r^2	.92	.98	.93
2			
Slope	1.22	1.12	1.06
Intercept	−49.20	−19.50	0.87
r^2	.90	.98	.99
3			
Slope	−0.15	0.23	1.22
Intercept	202.30	74.69	−24.60
r^2	.01	.11	.96
4			
Slope	1.04	0.95	0.92
Intercept	−0.26	20.88	36.90
r^2	.89	.93	.98
5			
Slope	1.05	0.88	0.81
Intercept	−8.55	18.17	33.39
r^2	.87	.98	.97
6			
Slope	0.92	0.72	0.75
Intercept	−40.10	12.03	36.98
r^2	.60	.91	.96
7			
Slope	−0.20	0.93	0.66
Intercept	205.77	30.66	47.71
r^2	.22	.86	.78
8			
Slope	1.10	0.98	1.02
Intercept	−12.17	−1.11	−10.54
r^2	.92	.93	.94

Table 8
Significance Levels of Beta Weights and Overall r^2 From Multiple Regression of α on Θ and L for Each Subject in Experiment 3

Subject	Θ	L	r^2
1	.0001	.27	.69
2	.0001	.23	.95
3	.12	.59	.17
4	.0001	.45	.92
5	.0001	.69	.92
6	.0001	.07	.76
7	.01	.63	.36
8	.0001	.54	.93

the h values found a significant effect of branch size, $F(2, 14) = 42.92$, $p < .001$. A Tukey test verified that the source of this difference was the contrast of the 6-cm branch with the two larger branches; the h measures for the 11.5- and 16.25-cm branches were not significantly different from each other. An overview of the data, with all circular statistics computed over subjects and responses, is presented in Table 10. (With respect to the proximity of α to Θ as measured by the modified Stephens procedure, analysis showed a significant difference with respect to 270° and 315° for the 6-cm branch only.)

Dependency of α on I and Θ. The 144 α values (18 for each of 8 subjects) were regressed on I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, and I_{yz} as given in Table 6. The multiple regression yielded $r^2 = .37$ ($p < .0001$) with the following significance levels: I_{xx}, $p < .01$; I_{yy}, $p > .05$; I_{zz}, $p > .05$; I_{xy}, $p < .02$; I_{xz}, $p < .0001$; I_{yz}, $p > .05$. The partial F values of these terms were, respectively, 6.42, 2.13, 1.57, 5.84, 28.67, and 0.27. The dependency of α on **I** is much reduced in the present experiment, and components found to be significant in Experiments 1 and 2 were not significant here.

The components-of-**I** hypothesis requires that **I** account for α at least as well as Θ accounts for α. Simple regression revealed that Θ accounted for 67% of the variance in α. The additional 30% of the variance accommodated by Θ over that accommodated by **I** suggests that the components-of-**I** hypothesis was not satisfied in the present experiment. Alternative hypotheses will have to be considered.

Eigenvectors and eigenvalues. As noted in the introduction, for any **I**, axes can be found such that the off-diagonal terms go to zero. In the diagonalized form of **I**, the axes are the eigenvectors or principal directions, and the magnitudes on the diagonal are the eigenvalues or principal moments of inertia. The tensor calculations yielding the values in Table 6 (and in Table 1) were conducted in an arbitrary set of rectangular coordinates through a fixed point O in the wrist (Figure 1d). For simplicity, x and z were assumed parallel to the ground plane with y perpendicular to the ground plane. The only nonarbitrary coordinate system with origin at O is that comprising the principal axes.

Table 9
Significance Levels of Beta Weights and Overall r^2 From Multiple Regression of h on Θ and L for Each Subject in Experiment 3

Subject	Θ	L	r^2
1	.23	.04	.31
2	.45	.04	.28
3	.52	.001	.53
4	.16	.001	.63
5	.22	.06	.28
6	.18	.01	.43
7	.67	.01	.34
8	.13	.27	.20

An alternative to the components-of-**I** hypothesis is that perceived orientation is constrained by the principal or symmetry axes (eigenvectors) of the L-shaped rod around O. Figure 7 shows the z eigenvectors for each orientation of each of the three L-shaped rods used in Experiment 3. These eigenvectors, together with the x and y eigenvectors, were obtained by diagonalizing the matrices given in Table 6 and are thus computed with respect to the arbitrary set of coordinates. As can be seen from inspection, the z eigenvectors for the different orientations are most closely aligned for the L-shaped rod with the 6-cm branch and least closely aligned for the L-shaped rod with the 16.25-cm branch. We can ask how well the eigenvectors account for the variance in α. Regressing the 144 α values on the x, y, and z eigenvectors, with each eigenvector expressed as three coordinate values, yielded $r^2 = .52$ ($p < .0001$) with 7 of the 9 coordinates significant, after backward elimination, at $p < .05$ or better. The eigenvectors account for more variance than **I**, suggesting that the significance of **I** lies in the eigenvectors.

Recognizing that diagonalizing **I** produces two sets of quantities, the principal directions and the principal moments of inertia, the preceding analysis suggests that perceived orientation may be independent of the latter set, the eigenvalues. Multiple regression of α on the eigenvalues showed no significant dependency, $r^2 = .02$ ($p > .05$). Despite these outcomes in favor of the eigenvector hypothesis, Θ was the better predictor of α. The eigenvectors accounted for 15% less variance. This failure of the eigenvector hypothesis is only apparent, however, and tied to the incommensurability between the dependent measure of α and the L-shaped objects' eigenvectors.

A vector response measure. In Experiment 3, as in the other experiments of the present article, the subject oriented a pointer on a dial to report his or her impression of the direction in the xy plane in which the branch of the L-shaped object was pointing. This report was then recorded as an angle. It is now important to recognize that the angle measure is derivative. From the subject's perspective, it may be supposed that the act of wielding provides an impression of the direction of the tip of the branch from the axes of wielding in the wrist. That is, the subject's impression is "vectorial." The spatial location of a branch's tip would be (x, y, z) in the rectangular coordinate system at O. In Experiment 3 the coordinate z would be a constant, fixed by the common stem length of the three L-shaped rods. In contrast, the coordinates x and y would be different for the three different-size branches oriented the same way in the xy plane, say 45°.

The tip of each branch at each Θ was expressed as a coordinate pair (x, y). The coordinates were calculated with O the same mean perpendicular distance (7 cm) from the

Table 10
Mean Circular Statistics for the Three Branch Sizes and Θ in Experiment 3

		Statistic			
	Θ (deg)	α (deg)	*r*	*z*	*h*
6.0-cm branch					
	0	13.80	.30	1.64	.39
	45	33.46	.48	4.22	.48
	90	70.84	.63	7.14	.60
	135	105.60	.43	3.33	.38
	180	161.80	.45	3.64	.42
	225	222.00	.24	1.04	.24
	270	312.00**	.34	2.08	.25
	315	366.40*	.52	4.87	.32
11.5-cm branch					
	0	4.97	.78	10.95	.78
	45	73.68	.64	7.37	.56
	90	88.00	.89	14.26	.89
	135	142.40	.79	11.23	.78
	180	152.67	.80	11.52	.71
	225	230.00	.67	8.08	.67
	270	262.00	.66	7.84	.66
	315	329.00	.70	8.22	.68
16.25-cm branch					
	0	10.35	.82	12.10	.80
	45	78.29	.65	7.60	.55
	90	108.50	.94	15.90	.89
	135	131.40	.86	13.31	.86
	180	163.20	.93	15.57	.89
	225	245.00	.75	10.12	.70
	270	270.00	.88	13.94	.88
	315	301.00	.60	7.84	.64

$*p < 05.$ $**p < .01.$

longitudinal axis of the stem as in the calculations of the **I**s (see Figure 1d). The majority, therefore, were in the lower quadrants of the *xy* plane, that is, $(x, -y)$ and $(-x, -y)$. Multiple regression analysis showed that the actual magnitudes of *x* and *y* were predicted nearly perfectly by the eigenvectors. For each subject, the computation of the (x, y) coordinates of the branch tip was repeated for each branch length at each α, resulting in 144 perceived *x* coordinates and 144 perceived *y* coordinates (18 of each for each of 8 subjects). These perceived *x* and *y* coordinates were regressed on the actual coordinates and the eigenvectors. The regressions on the actual *x* and *y* coordinates yielded r^2 values of .72 and .59, respectively. The regressions on the eigenvector yielded r^2 values of, respectively, .76 (with the *x* and *z* of the *z* eigenvector, the *y* and *z* of the *x* eigenvector, and the *z* of the *y* eigenvector significant after backward elimination) and .61 (with the *x* and *y* of the *z* eigenvector and the *y* of the *y* eigenvector significant after backward elimination). To round out this analysis with the vectorial dependent measure, the eigenvectors or body axes of the L-shaped objects at each orientation were expressed in relation to the spatial axes anchored in the wrist (*Oxyz*) through Eulerian angles (see Goldstein, 1980). Multiple regressions of the perceived *x* and perceived *y* coordinates on the three Eulerian angles yielded r^2 values of .69 and .59, respectively.

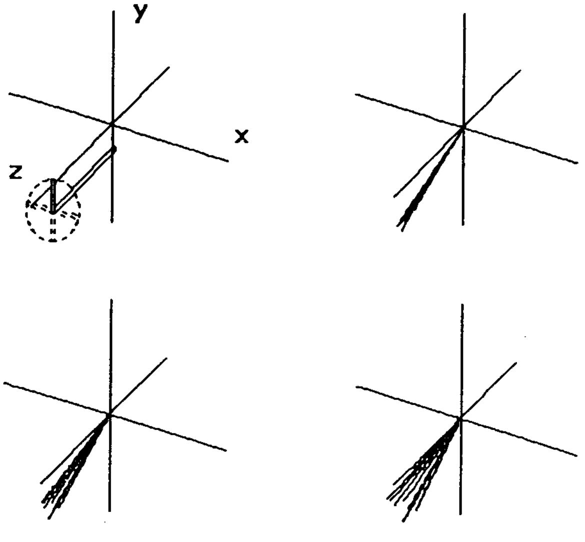

Figure 7. The upper left panel depicts the relation of the L-shaped rod to the arbitrary rectangular coordinate system with origin at *O*; the *z* eigenvectors for the 6-cm, 11.5-cm, and 16.25-cm branched rods are depicted in the upper right, lower left, and lower right panels, respectively.

The conclusion to be drawn is that perceived *x* and *y* are predicted by both actual *x* and *y* and the eigenvectors to the same degree. The latter, in a different guise, is the comparison between Θ and the eigenvectors as predictors of α. The eigenvectors are directions from *O* and may need to be matched with a perceptual measure that is a direction of like kind; α does not fit the bill. When the subjects' αs are converted into vectors originating at *O*, the results of Experiment 3 are seen to be consistent with the eigenvector hypothesis.

General Discussion

In the experiments of the present article, we have entertained the hypothesis that if the muscular and tendon deformation consequences of wielding an L-shaped rod are specific to **I** (or some aspect of **I**) and are detectable, then the direction in which the branch of an L-shaped rod points is perceptible by dynamic touch. The results of the three experiments are consistent with this hypothesis. In all three experiments, perceived orientation was a linear function of actual orientation. Moreover, the parameters of the functional dependence of α on Θ were independent of style of wielding and size of object. What did vary with these independent manipulations was the accuracy of perceived orientation as measured by *h*. Given that Θ per se cannot affect the mechanoreceptive tissues of the body, and given that **I** can, then the dependence of α on Θ is understandable as a dependence of α on **I**. On the basis of Experiment 3, it must be assumed that the dependency of α on **I** goes deeper than the particular componential values of **I**. Analysis suggests that α, appropriately interpreted as a vector in *Oxyz*, where *O* is the origin at the fixed point of rotation and *xyz* are the coordinate axes, may be specific to the eigenvectors of **I**. To evaluate more fully this eigenvector hypothesis, future experiments will have to use objects different from those of the present experiments, specifically objects for which the covariation between spatial orientations (expressed as vectors in the space axes *Oxyz*) and eigenvectors (the symmetry axes of the mass distribution) can be broken.

The hypothesis investigated in the present article, and the more specific hypothesis developed as a result of the investigation, are continuous with a more general hypothesis that perception is specific to information. This general information-specificity hypothesis goes hand-in-hand with the hypothesis that information is specific to properties of the environment and self-movement (Gibson, 1959, 1966, 1979). In Gibson's (1979; Reed & Jones, 1982) ecological approach to perception, information is defined through laws (Kugler & Turvey, 1987, 1988; Turvey, 1990; Turvey, Shaw, Reed, & Mace, 1981), with *law* understood in the conventional sense as an invariant relation among properties of things (Bunge, 1977). For visual perception, the invariant relations are among properties of the environment and perceiver movement and properties of distributions of light energy in air or water. For haptic perception, the invariant relations are among properties of the environment and perceiver movement and properties of distributions of mechanical energy in biological tissues. Customarily, within the ecological approach, the term *information* is ascribed to the properties of the distributions satisfying these invariant relations. They are said to be information about (in the sense of *specificity to*) the environmental and self-movement properties. As Gibson (1979; Reed & Jones, 1982) expresses it, optical information is contained in structured arrays consisting of different intensities, or different rates of change in intensity, in different directions. Similarly, mechanical information is contained in structured arrays consisting of different tissue strains, or different rates of change in tissue strain, in different directions. Mathematically, information is contained in tensors, given that tensors are quantifications of structured arrays.

In the case of an L-shaped object wielded around a fixed point in the wrist, the muscular and tendon deformations are causally entailed by the torques $\mathbf{N} = \mathbf{I}\cdot(\dot{\omega}) + \omega \times (\mathbf{I}\cdot\omega)$ governing the three-dimensional motions of the hand and object (where $\mathbf{I}\cdot[\dot{\omega}]$ is the contribution to the variation in angular momentum arising from variation of the angular velocity components, and $\omega \times [\mathbf{I}\cdot\omega]$ is the contribution due to the rotation of the principal axes to which the angular momentum is referred) (see Euler's equations in Goldstein, 1980; Symon, 1971). Although $\mathbf{N}$, $\dot{\omega}$, and ω will vary in the course of wielding a given oriented L-shaped object during a trial and across trials, $\mathbf{I}$ is constant and determined by the time independent L-shaped object and its fixed orientation in the hand. The structured array of tissue deformations would be quantified (presumably) by the strain tensor $\mathbf{u}$ or rate of strain tensor $\mathbf{v}$. On the law-based approach to defining information, information about orientation would be available if time-independent aspects of $\mathbf{u}$ or $\mathbf{v}$ related invariantly to $\mathbf{I}$ (Solomon, 1988; Solomon & Turvey, 1988; Solomon et al., 1989a, 1989b), more particularly, if a property of the tissue deformations related invariantly to an orientation-specific property of $\mathbf{I}$ (say, to its eigenvectors). On the information–perception specificity hypothesis, if the aforementioned deformation property is detectable, then perception will be specific to the orientation-specific property of $\mathbf{I}$ and thereby to the direction in which the branch of an L-shaped rod points. These arguments echo Gibson's (1966, p. 127) remark that "the stimulus information from *wielding* can only be an invariant of the changing flux of stimulation in the muscles and tendons, an exterospecific invariant in this play of forces."

It has been argued that the detection of invariants underlies the perception of rod orientation through dynamic or effortful touch. The fact remains, however, that in the research reported here, subjects exhibited different levels of sensitivity under different wielding conditions and with objects of different magnitudes. Namely, the homing tendency decreased when subjects were restricted to wielding in only one plane (Experiment 2) and when branch length was decreased (Experiment 3). That sensitivity differences arise in cases in which invariants may underlie perception is an issue to be addressed in future research.

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