# Autonomous and Nonautonomous Dynamics of Coordinated Rhythmic Movements

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Recently, accounts of human rhythmic movement have been given in terms of nonlinear dissipative dynamics with special emphasis on the autonomous dynamics that characterize self-organizing systems. In this article, an argument is made for a modeling strategy that allows for and, when possible, attempts to capitalize on the incorporation of nonautonomous dynamics in models of human rhythmic movement, that is, dynamics that characterize systems that are forced externally. The problem is to incorporate these nonautonomous dynamics nonarbitrarily. Cascade juggling and swinging hand-held pendulums provide the coordinated rhythmic movements to which the argument is applied. Empirical and theoretical analyses suggest that the dynamical description of the movements composing these activities may be nonautonomous when the level of analysis is restricted to the motions of the subsystems in isolation; there are small but systematic forcing contributions in addition to the dominant autonomous components. It is suggested, however, that autonomy is restored when the level of analysis is raised from that of the motions to that of the perception–action cycles, so that dynamically based informational terms are included. At this level, forcing is a function of information, not clock time. A strategy for investigating coordinated rhythmic movements is proposed in which (a) the full complement of tools from nonlinear dynamics is put to use, (b) the determination of nonautonomous components follows the evaluation of autonomous components, and (c) informational variables are sought to transform the resultant dynamical description of the coordinated activity to full autonomy.

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In this article we address issues of contemporary importance in the study of human movement coordination. Our primary focus is on the developing nonlinear dynamical approach. To date, this approach to coordinated movement has emphasized the dynamics associated with systems that are self-organizing and self-sustaining rather than externally driven (e.g., Kay, 1986; Kay, Kelso, Saltzman, & Schöner, 1987; Kugler, Kelso, & Turvey, 1980, 1982; Kugler & Turvey, 1987; Saltzman, 1986; Schöner & Kelso, 1988). Nonautonomous descriptions of coordinated movement patterns, that is, descriptions in which the flow of the system depends explicitly on time, have been discarded in favor of autonomous descriptions, that is, descriptions in which the flow of the system only depends on the system's internal (or state) variables (i.e., position and velocity).

Our proposal in this article is for a thoroughgoing incorporation of all aspects of nonlinear dynamics without making the a priori assumption that rhythmically coordinated movement systems are always and necessarily autonomous, irrespective of how their boundaries are defined. We argue that this full-fledged dynamical approach is needed to accommodate dynamical systems, such as biological systems, that are governed more by information than by forces (see Kugler & Turvey, 1987). The strategy of research and theory allied to this proposal suggests to first describe the motions of the movement subsystems of perception–action systems in terms of their autonomous and nonautonomous dynamics that together comprise the dynamical basis of the coordinated movement in question (P. J. Beek & W. J. Beek, 1988; P. J. Beek, 1989a, 1989b), with the identification of autonomous components preceding and constraining the identification of feasible nonautonomous components. Subsequently, it is suggested to restore the autonomy of the description by raising the level of analysis from that of the motions to that of perception–action cycles and substituting the nonautonomous components by informational variables that capture the coupling of the movement subsystems among each other and to the environment. A major challenge our proposal faces is that of including both the autonomous and the nonautonomous components in physically well-motivated, nonarbitrary ways. As we hope will become apparent, our efforts to meet this challenge bear on the conventional motor programming formulations of movement coordination and, more importantly, on the roles played in movement coordination by intention and information.

## BIOMECHANICS AND MOTOR PROGRAMS

In the study of the coordination and control of movement it has been customary to distinguish the concerns of the biomechanicist from those of the neuroscientist and psychologist. In biomechanics one observes a movement pattern and attempts to formulate an account of certain aspects of its spatio-temporal

details (its kinematics) in terms of muscular and reactive forces (its kinetics). In neuroscience and psychology one observes a movement pattern and attempts to formulate an account of certain aspects of its spatio-temporal details in terms of neural and/or mental processes precipitating the muscular and reactive forces. The two accounts are largely complementary, with biomechanics addressing those aspects that follow from the laws of mechanics, and neuroscience and psychology addressing those aspects that do not. Obviously the biomechanical account is tightly constrained; the laws of mechanics cannot be compromised. In contrast, the complementary account given by neuroscience and psychology is loosely constrained. In the absence of laws in the strict sense of the word, constraints on this latter account are limited to the criteria of sound logic, applied within the context of the prevailing model of internal processes, and by the data of experiments, conducted and analyzed within that context. The account must be at least consistent with the premises of the model.

The need to affix an account in terms of internal processes to an account in terms of the laws of mechanics arises primarily from the conceptual separation expressed roughly as that between mind or brain and body. The laws of mechanics apply only to the body. The mind/brain sets the body segments into motion and identifies basically what form the variously directed motions of the segments should take. A most significant feature of coordinated movement is that the adjacent and successive order of the skeletomuscular variables and their various magnitudes appear to be anticipated. That is, coordinated movements appear to be programmed. In many quarters a major challenge for a thoroughgoing science of movement—perhaps the major challenge—is understanding this programming. A widespread opinion is that the programs of movement coordination are expressed in a language that is quite different from the language of natural laws. For example, suitable formulations of motor programs seem to be in terms of symbol strings, or weighted connections among processing units, where these formulations embody essentially a model or representation of the movement.

## UNDERSTANDING COORDINATION THROUGH NONLINEAR DISSIPATIVE DYNAMICS

In recent years there has developed an approach to the study of movement coordination aimed at understanding all of its aspects in law-based terms. The approach might be called the dynamical systems approach to movement coordination. The basic hypothesis is that the phenomena of movement are understandable as the outcomes of nonlinear dissipative dynamics (e.g., P. J. Beek, 1989a; Kay, 1988; Kay, Saltzman, & Kelso, 1991; Kugler & Turvey, 1987; Schöner & Kelso, 1988; Turvey, 1990a, 1990b). This hypothesis extends the physical analysis of movement beyond the laws of classical mechanics to the

principles and laws of self-organizing, complex systems. The significance of this extension is that an analytic strategy exploiting the principles and laws in question ought to bear simultaneously on the internal processes and the bodily motions. The promise is a softening of the classical separation, and an account of coordination's basis—the aspect of particular interest to neuroscience and psychology—in a language that is intimately connected to dynamics.

In large part, but by no means exclusively, the nonlinear dynamical perspective on movement coordination was fostered and advanced by the ecological approach to perception and action, with its emphasis on general principles and its deemphasis of specific computational–representational formulations (e.g., Carello, Turvey, Kugler, & Shaw, 1984; Gibson, 1979; Michaels & Carello, 1981; Turvey & Shaw, 1979; Turvey, Shaw, Reed, & Mace, 1981). From an ecological point of view, proposed motor programs as computational-representational entities seemed to fall prey to excesses of detail and arbitrariness of form (Fitch & Turvey, 1978; Fowler & Turvey, 1978; Kelso, 1981; Turvey, Shaw, & Mace, 1978). The expectation was that, given a physical tack on the problem of coordination, one that respected biology's nonlinear dissipative nature, the abstractly defined processes or structures underlying coordinated movements would be revealed as sparse in detail (technically speaking, of low dimension) and shaped nonarbitrarily by dynamics (Kugler et al., 1980, 1982; Turvey, 1977).

## AUTONOMOUS AND NONAUTONOMOUS DYNAMICS

A system's dynamics can be represented by either autonomous or nonautonomous differential equations. An autonomous equation contains no terms in which time $t$ (the independent variable) is involved explicitly; a nonautonomous equation, in contrast, does involve time explicitly. An autonomous system is defined by the time-indifferent state equation:

$$dx/dt = f_t(x), x(t_0) = x_0 \qquad (1)$$

and a nonautonomous dynamical system is defined by the time-varying state equation:

$$dx/dt = f_t(x,t), x(t_0) = x_0 \qquad (2)$$

To choose two specific examples, the equation

$$m(d^2x/dt^2) + b(dx/dt) + kx = E(\phi) \qquad (3)$$

is autonomous because the argument $\phi$ of function $E$ is a state variable of the system (viz., the phase angle in the $x - dx/dt$ phase plane), whereas the equation

$$m(d^2x/dt^2) + b(dx/dt) + kx = A\cos(\omega t) \qquad (4)$$

is nonautonomous because it depends explicitly on the external "clock" time, $t$. In the example of the nonautonomous equation, a linear mass-spring system of mass $m$ with damping and stiffness coefficients $b$ and $k$, respectively, is controlled by a time-independent forcing function, $A\cos(\omega t)$, which is independent of the system's state. In the example of the autonomous equation, the function $E(\phi)$ governs the injection of energy to offset the energy lost through the friction term, $b(dx/dt)$, and is dependent on the value of the mass-spring system's kinematic variable of phase angle, $\phi$. In the autonomous case, the initial time may always be taken as $t_0 = 0$, which is not permitted in the nonautonomous case. The essential aspect of a nonautonomous system is that the clock variable affects, but is unaffected by, the rest of the system variables.

Within the dynamical perspective, a marked preference has been expressed for conceptualizing coordinated rhythmic movements in terms of autonomous dynamics rather than nonautonomous dynamics (Kay, 1986; Kay, Kelso, Saltzman, & Schöner, 1987; Kugler et al., 1980; Kugler & Turvey, 1987). Two major reasons have been put forward. The first reason is the need to incorporate explicitly into the understanding of coordination the self-organizing, self-sustaining processes characterizing the class of open, dissipative systems, of which humans and animals are members. Self-organizing and self-sustaining dynamical processes are best expressed in autonomous form (Haken, 1977, 1983; Yates, Marsh, & Iberall, 1972). The second reason for advocating autonomous dynamics over nonautonomous dynamics is the need to halt the regress of control structures. With respect to rhythmicity, for example, it has been argued that if there is a cyclic forcing function, independent of a rhythmically moving system, then we must ask the question, what is the forcing function's timekeeper? Implicating another forcing function and, hence, additional set-points for control renders the analysis of rhythmic movement regressive and unenlightening.

The forcing functions characterizing nonautonomous dynamics, external as they are to the systems they force, bear close conceptual similarity to motor programs (cf. R. A. Schmidt, 1988), which are said to be logically separate from, and hierarchically superior to, the subsystems that are coordinated. Consequently, within the general ecological approach it has been felt that any attempt to model coordination strictly through nonautonomous dynamics is not likely to fare any better than motor programming formulations. Such forcing functions can be proposed arbitrarily to accommodate any of the various details of coordinated activity.

However, equating the physical notion of nonautonomy with the psycholog-

ical notion of motor programs qua time-keeping mechanisms is problematic. When distinctions drawn from different disciplines meet, one has to be extremely careful not to let the technical meaning of one distinction (i.e., autonomy vs. nonautonomy) be dominated or corrupted by the technical meaning of the other distinction (i.e., motor programming or not). Dynamically speaking, a particular system is either an autonomous or a nonautonomous system with the sole criterion of whether physical time appears explicitly in the system's equations of motion. Whether the nonautonomous dynamics can be equated with a motor program or not is quite another issue. In this context, it is crucially important to realize that the autonomous–nonautonomous distinction is just as susceptible to one's selected frame of reference as is the psychological distinction between emergent and preprogrammed timing (cf. Saltzman & Munhall, 1989). The reason is that any nonautonomous system of equations can be transformed into an autonomous one by adding one or more equations describing the dynamics of the (formerly) external clock-time variable. That is, the system boundaries of the system definition can be extended to include the dynamics of both the original nonautonomous dynamics as well as the (formerly) external clock. Equation 4, for instance, which is a nonautonomous second-order equation, can be converted into a set of three autonomous first-order equations if one defines $dx/dt = y$ and employs the standard trick of regarding $t$ as one of the variables with the third equation simply $dt/dt = 1$. In this new set of equations, a state of unilateral coupling exists between system elements. Conversely, any autonomous system of equations can be transformed into a nonautonomous one by adding one or more equations describing the (formerly) autonomous dynamics of a conveniently chosen subpart of the original system as acting nonautonomously on some other conveniently chosen subpart of the original system, which now has become the system. That is, the system boundaries of the system definition can be narrowed down to exclude a part of the (formerly) autonomous dynamics of the system. If the two subsystems of the original system were coupled unilaterally, then this move does not affect the validity of the description. However, if the two subsystems of the original system were coupled bilaterally, then the system description looses accuracy. Because the issue of autonomy–nonautonomy depends on the definition of the boundaries of the system under study, the psychological interpretation of nonautonomous dynamics as being necessarily equivalent to motor programs in terms of internal representations responsible for time keeping is unwarranted. All that is implied by a nonautonomous system is that the composition of the system in question is such that it can be conceptualized as a system that is externally and unilaterally forced by another system or set of systems, whose forcing actions can be modeled in a time-dependent way. This need not imply an infinite regress, nor the existence of internal representations.

In contrast to the psychological interpretation of nonautonomous dynamics, we emphasize here the need to view the incorporation of nonautonomous forcing functions as a strictly operational step on the part of the scientist in seeking an optimal characterization of the system under study. As such, incorporating nonautonomous dynamics provides, as we argue, a potentially useful tool for uncovering the informational/intentional basis of coordinated rhythmic movements.

## CAN NONAUTONOMOUS DYNAMICS BE INCORPORATED IN A NONARBITRARY WAY?

In the context of the ecological critique of motor programming, the distinction between arbitrary and nonarbitrary constructions of abstract control spaces—the intentional contents of motor actions—is crucial. In the ecological critique, the qualifier *arbitrary* refers to constructions that are discontinuous with the strategy of understanding phenomena through the careful extension of natural laws. Arbitrary constructions invoke concepts that are neither physical in themselves nor constrained by general physical principles. Arbitrary constructions of intentional content comport with a deep-rooted assumption that intentionality is a psychological category par excellence and, thus, per definition, devoid of principles of a more general, physical nature. By contrast, the qualifier *nonarbitrary* in the ecological critique refers to constructions that are completely continuous with natural law. Ideally, these constructions invoke concepts that are either physical in themselves or so constrained by physical principles that it is evident how intentional content connects with the dynamics that it harnesses (see later in this article).

Returning to the issue of nonautonomous control dynamics in accounts of coordination, we now note that nonarbitrary routes to understanding how coordination is created within the perception–action cycle may well include the identification and exploitation of a nonautonomous "timer," without making the assumption that this timer need also be represented internally, other than in the combined effect of a set of dynamical subsystems. The only requirement would be that its inclusion satisfies continuously the criterion of nonarbitrariness. The ecological critique of motor programming attempts to dispense with all arbitrary forms of nonautonomous components, and to motivate nonarbitrary forms of nonautonomous components. The question, therefore, is whether or not nonautonomous dynamics can be incorporated in a nonarbitrary way.

## THE *W*-FUNCTION

Generally speaking, it is hard to distinguish between autonomous and nonautonomous systems solely on the basis of the kinematic measures ([angular]

position and [angular] velocity) of a movement cycle. For instance, if the phase-plane representation of position against velocity exhibits a steady state or a limit set that is a perfect circle, then the nature of the underlying dynamics may be autonomous as well as continuously forced, and there will be no way to distinguish between the two on topological grounds alone. It is extremely unlikely, however, that kinematic measurements of the rhythmic movements of animal or human will, if ever, produce perfect circles in the phase plane. In many cases, they tend to deviate from the circle in nontrivial ways. Moreover, the trajectories of multiple cycles plotted in a phase plane typically intersect. Figure 1 shows several phase portraits of a particular rhythmic movement: the comfortable swinging of a hand-held pendulum through motions at the wrist for an interval of 15 s to 20 s (see Kugler & Turvey, 1987; Rosenblum & Turvey, 1988; Turvey, Rosenblum, R. C. Schmidt, & Kugler, 1986; Turvey, R. C. Schmidt, Rosenblum, & Kugler, 1988). The phase portraits are of such wrist-pendular activity in the context of the subject moving rhythmically two pendulums, one in each hand, at the same average frequency (1:1 frequency locking in the mean). Each phase portrait in Figure 1 depicts the behavior of one of these rhythmic units. In each case, the trajectories in the phase plane intersect and the resultant orbital patterns deviate from a circle in more or less obvious ways. The observed differences in orbital shapes are associated with differences in the conditions of 1:1 frequency locking. The two hand-held pendulums can differ in size (mass and length) with the result that, for comfortable 1:1 coordination pattern, the smaller pendulum must move slower than its characteristic frequency and the larger pendulum must move faster than its characteristic frequency (see Kugler & Turvey, 1987; R. C. Schmidt, P. J. Beek, Treffner, & Turvey, 1991; Turvey, R. C. Schmidt, & P. J. Beek, in press, for details). What we focus on here is the fact that trajectory intersections in the phase plane and orbital deviations from circularity of the kind identified in Figure 1 may provide some first clues as to the nature of the underlying dynamics.

With regard to trajectory intersections in the phase plane, there is a well-known fact concerning the existence and uniqueness of solutions to the dynamical systems described by Equation 1 and Equation 2 that is important in this context (cf. Parker & Chua, 1987). Assume that for any time $t$, $f_t$ is a diffeomorphism (meaning that $f_t$ and $f_t^{-1}$ are differentiable at each point, and both are single valued and continuous). For systems represented by Equation 1, that is, for autonomous systems, $f_t(x)$ equals $f_t(y)$ if and only if $x = y$, implying that the trajectories are uniquely prescribed by the system's state at a given time. When projected in a fully defined state space, that is, a state space that has as many dimensions as the system it is a space of, the trajectories of these systems cannot intersect. The trajectories can appear to cross, however, when viewed as projections into a lower dimensional subset of the state space. For systems represented by Equation 2, $f_t(x, t_0)$ equals $f_t(y, t_0)$ if and only if $x = y$. The latter equation implies that, given the initial time, a trajectory of a nonautonomous

system is specified uniquely by the initial state with the qualification that it is possible that $f_t(y, t_0)$ equals $f_t(y, t_1)$ if $t_0 \neq t$ and $x \neq y$ showing that the trajectories of nonautonomous systems can intersect in a state space with as many internal dimensions as the system in question. Only when time is added as an additional dimension do the crossings disappear. The fact that the trajectories of multiple-movement cycles projected in the phase plane intersect is evidence that more than two dimensions are needed to describe the dynamics of the system. It is possible that the system in question is a second-order nonautonomous system and that the dimension to be added is external time. It is equally possible, however, that the dynamics are autonomous and of dimension greater than two. In the case of a bimanual wrist-pendulum, for instance, it is reasonable to include the state variables of the second wrist-pendulum system in a set of coupling terms in the equation of motion of the first wrist-pendulum system (and vice versa), so as to model the system as two mutually coupled nonlinear oscillators (Turvey et al., in press). In addition to higher order autonomous dynamics and nonautonomous dynamics, noise may be the cause of the trajectory crossings in the phase plane. (If one considers noise as an extrinsically imposed, infinite-dimensional process, this possibility collapses with the possibility that the dynamics are autonomous and of dimension greater than two.) Thus, nothing can be said vis-à-vis the autonomy–nonautonomy distinction on the basis of the existence of trajectory crossings other than that the system is autonomous and noise-free when trajectory crossings are absent in a fully defined state space. A detailed analysis of how the trajectories cross each other, however, might be very revealing as to the underlying dynamics: Intersections that are nearly tangential to some average trajectory indicate dynamics very different from intersections that are nearly normal to average.

Second, the topology of the limit set itself may provide information about the underlying dynamics. It may, for instance, deviate from the circle in such a way that it is hard to reconcile with the state space of an autonomous oscillator. To substantiate this claim, we summarize the results of an analysis offered by P. J. Beek and W. J. Beek (1987, 1988). On the basis of topological considerations, P. J. Beek and W. J. Beek (1987, 1988) provided a mathematical argument that four elementary series are sufficient to describe the conservative and non-conservative terms of second-order autonomous oscillators. For the conservative terms, the relevant series are the Duffing series $(x, x^3, x^5, \ldots)$ and what might be termed the $\pi$-mix odd series $[x(dx/dt)(dx/dt), x^3(dx/dt)^3(dx/dt), \ldots]$. For the nonconservative terms, the relevant series are the Van der Pol series $[x^0(dx/dt), x^2(dx/dt), x^4(dx/dt), \ldots]$, the Rayleigh series $[(dx/dt)^0(dx/dt), (dx/dt)^2(dx/dt), (dx/dt)^4(dx/dt), \ldots]$, and what might be termed the $\pi$-mix even series $[x^2(dx/dt)^2(dx/dt), x^4(dx/dt)^4(dx/dt), \ldots]$. The conservative terms exhibit properties that tend to influence oscillatory frequency more than amplitude; they tend not to affect the total energy balance of the limit cycle. The nonconservative terms, in contrast, influence oscillatory amplitude more so
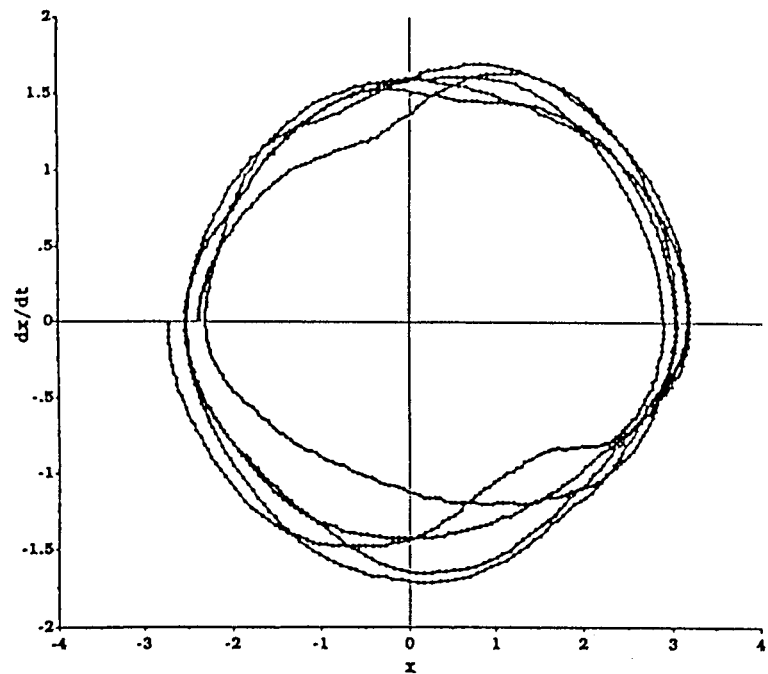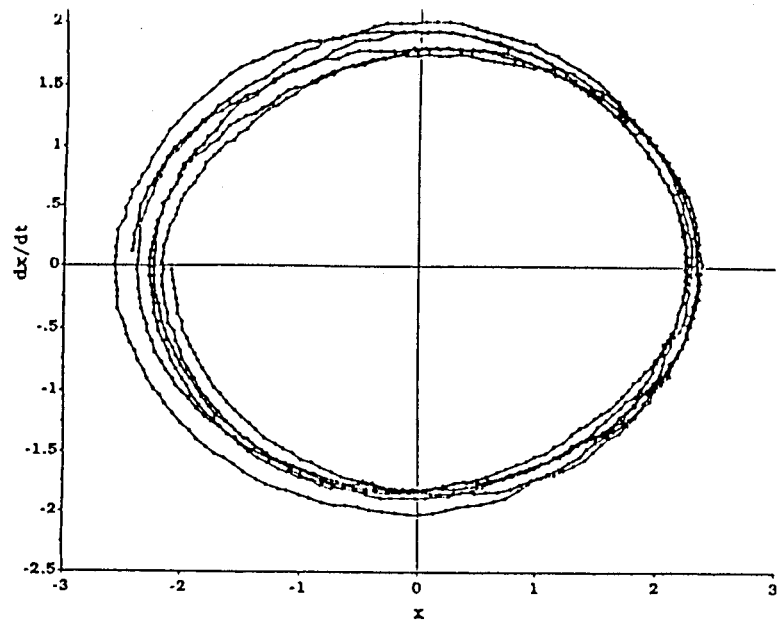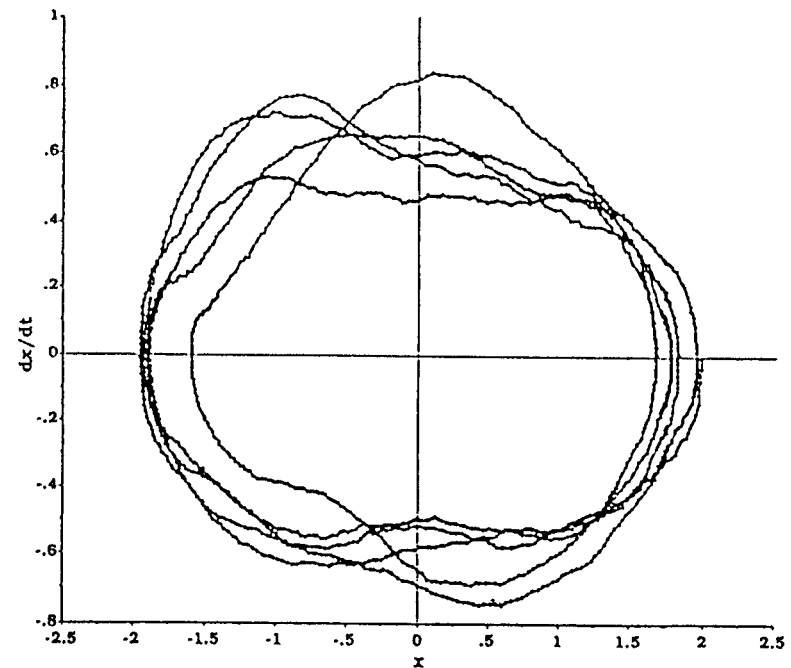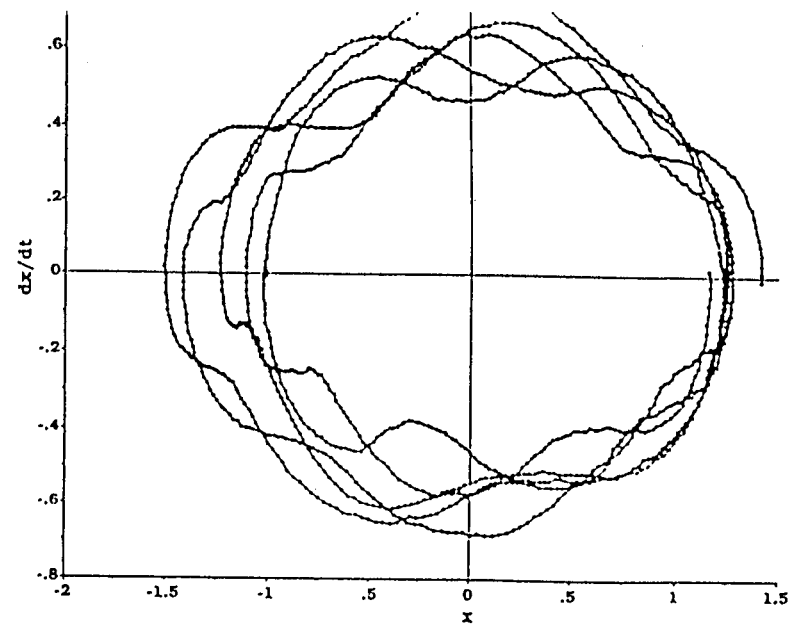
(continued)

FIGURE 1 Examples of phase portraits of wrist-pendulum system activity indicating intersection of trajectories and limit cycles that deviate in different ways from the circle. $x$ is angular displacement (in rad normalized to the frequency of the oscillator in Hz) and $dx/dt$ is angular velocity (in rad/s). (a) is a system oscillating in the uncoupled state. (b), (c), and (d) are examples of systems participating in 1:1 frequency locking. With the depicted system's eigenfrequency given by the numerator, the ratios of the uncoupled frequencies in the three coupled cases were: 1.27/.74, .57/1.27, and .57/1.30.

than frequency; they tend to affect the total energy balance of the limit cycle. Basically, the $\pi$-mix series is the product of the Van der Pol and Rayleigh series. Its significance is that it identifies a new class of conservative and non-conservative terms for oscillators with a single degree of freedom.

From the P. J. Beek and W. J. Beek (1987, 1988) catalogue of functions, useful analytical and graphical tools can be derived for identifying the essential terms of nonlinear differential equations. The series are developed in a function $W(x, dx/dt)$ that reflects the continuous deviation of the limit set from the circle defined by $d^2x/dt^2 + \omega_0^2x = 0$:

$$d^2x/dt^2 + \omega_0^2x + W(x, dx/dt)\omega_0 = 0. \tag{5}$$

Multiplying by $dx/dt$ and integrating yields

$$W(x, dx/dt) = -d[(dx/dt)^2 + \omega_0^2x^2]/2d\omega_0x. \tag{6}$$

The $W$-function can be constructed once a reasonable estimate of the equilibrium position, which has to be assumed to be constant throughout a trial, has been made. $W(x, dx/dt)$ can be inspected graphically to scout locally for Duffing, Van der Pol, Rayleigh, and $\pi$-mix behavior, in anticipation of the fact that the local prominences of these basic oscillatory mechanisms change over different phases of the cycle. To scout locally for Duffing behavior, $W(x, dx/dt)$ is plotted as a function of $x$; for Rayleigh behavior, $W(x, dx/dt)/dx/dt$ is plotted as a function of $dx/dt$; for Van der Pol behavior, $W(x, dx/dt)/dx/dt$ is plotted as a function of $x$; for $\pi$-mix, $W(x, dx/dt)/x$ is plotted as a function $dx/dt$ (see P. J. Beek, 1989b and P. J. Beek & W. J. Beek, 1988, for further details). We present in Figure 2 an example of this graphic technique with respect to the pendular clocking mode of movement organization. The example is that of scouting for Duffing behavior in one of the cycles presented in Figure 1. As can be seen, the cycle of wrist-pendular activity depicted exhibits a cubic ($x^3$) relation between $W$ and $x$ that is of different sign for the two segments of the cycle. In the depicted cases, the cycles comprise both a hardening spring (the restoring force is exponentially larger than linear) in the course of ulnar flexion that closes the wrist angle (adduction), and a softening spring (the restoring force is exponentially smaller than linear) in the course of radial flexion that opens the wrist angle (abduction). It is also apparent from inspection that stiffness functions of higher order ($x^5$, $x^7$) may apply. An estimate of the coefficients of each of these Duffing terms (i.e., $x$, $x^3$, $x^5$, etc.) can be obtained by entering them in a multiple linear regression on $W$, performed either for each individual (half) cycle or for a bout of cycles. A measure of the "detuning," that is, the degree to which the wrist-pendulum system moves faster or slower than its eigenfrequency, can simply be obtained by estimating the slope of the linear regression of $x$ on $W$, which is a measure of the overall stiffness. From the fact that the slope of the
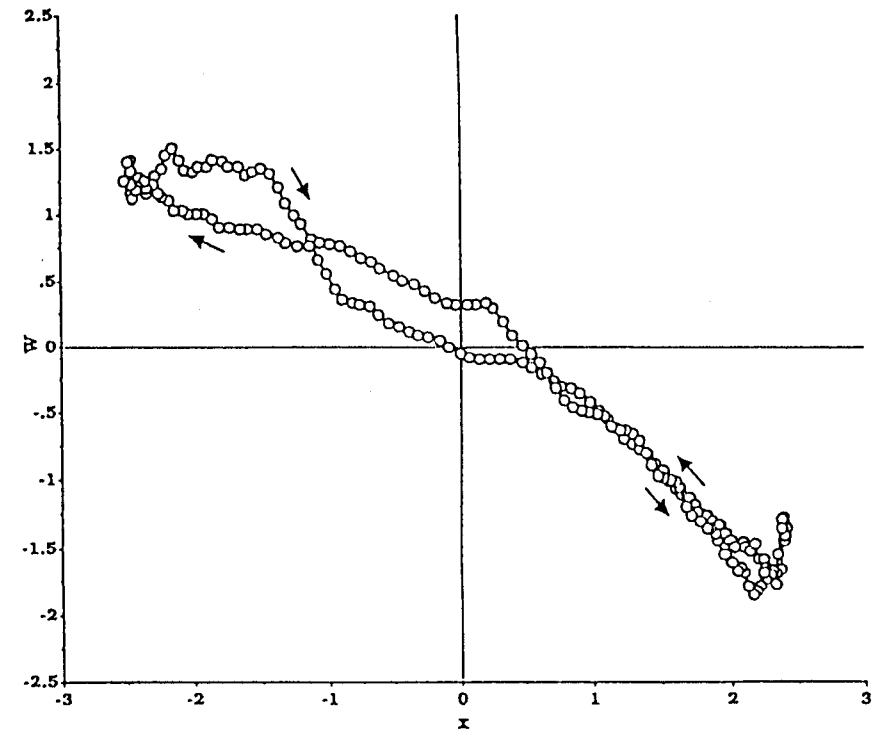


FIGURE 2 The graphic procedure for exhibiting detuning and the presence of Duffing terms in oscillatory behavior. The magnitude of detuning is indexed by the slope of the function relating $W$ to $x$; the presence of Duffing terms is indicated by the curvature of the function. It can be seen that the curvature of the function for radial flexion (right-pointing arrows) is positive, indicating a hardening spring, and the curvature for ulnar flexion (left-pointing arrows) is negative, indicating a softening spring. (See text for details.)

linear relation between $x$ and $W$ is negative, it follows that, during the selected cycle, the system was running slower than its eigenfrequency.

When these regression techniques are used to obtain estimates of the coefficients of all the terms (up to a certain power) in the identified series expansions, important insight into the dynamics can be gained. Table 1 shows the results of this approach for one particular subject swinging three different single wrist-pendulum systems for 12 complete cycles (defined from full abduction to full abduction). For each completed cycle, estimates were obtained for the coefficients of $x$, $dx/dt$, $x^3$, $x(dx/dt)^2$, $(dx/dt)^3$, and $x^2(dx/dt)$ during that cycle by regressing these variables on $W$ (using a stepwise regression technique). As can be seen from the results, the nonconservative terms prove to be significant more often in the smaller wrist-pendulum system than in the larger wrist pendulum, which is predominantly conservative. It can also be seen that there is, in

TABLE 1
Statistically significant parameter coefficients of the conservative (c) and nonconservative (nc) terms in the W-function as assembled during 12 cycles of swinging three single wrist-pendulum systems of different length L (where L = moment of inertia/static moment). Amount of variance in W accommodated on each cycle by the two classes of terms is identified by $R^2$.

| $R^2$ | x (c) | dx/dt (nc) | $x^1$ (c) | $x^2$(dx/dt) (nc) | $(dx/dt)^3$ (nc) | $x(dx/dt)^2$ (c) |
|---|---|---|---|---|---|---|
| **L = .226 m** | | | | | | |
| .947 | | .049 | 1.292 | | | −.044 |
| .977 | | | 1.500 | .209 | | −.053 |
| .981 | −.750 | | 2.027 | .162 | | |
| .956 | | | .911 | .162 | | −.033 |
| .945 | −.776 | | 2.088 | .259 | | |
| .898 | | .043 | 1.283 | | | −.048 |
| .930 | | | 1.164 | .193 | | −.039 |
| .836 | | .050 | .941 | | | −.029 |
| .950 | | .134 | .974 | | −.006 | −.030 |
| .865 | | | 1.197 | | .003 | −.044 |
| .899 | | | 1.348 | .173 | | −.049 |
| .990 | | .026 | 1.138 | .137 | | −.042 |
| **L = .377 m** | | | | | | |
| .870 | | .124 | 6.008 | | | |
| .920 | | .149 | 6.309 | | | |
| .876 | | .107 | 6.369 | | | |
| .935 | | | 7.720 | 1.386 | | |
| .982 | 5.388 | .116 | −15.260 | −.562 | | −.668 |
| .993 | 3.390 | .126 | −7.543 | | | −.353 |
| .879 | 1.431 | .165 | | | | |
| .934 | 1.416 | .103 | | | | |
| .927 | | .116 | 5.624 | | | |
| .941 | | .134 | 4.630 | | | |
| .995 | 2.947 | −.571 | −4.567 | 2.804 | .076 | −.266 |
| .950 | | .100 | 4.858 | | | |
| **L = .473 m** | | | | | | |
| .867 | .916 | | | | | |
| .983 | 1.073 | .184 | | −.585 | | |
| .836 | 1.339 | .123 | | | | |
| .912 | 1.485 | .126 | | | | |
| .911 | | .115 | 7.244 | | | |
| .934 | 1.584 | .123 | | | | |
| .925 | 1.412 | .168 | | | | |
| .952 | | .136 | 9.986 | | | |
| .984 | 1.915 | .184 | | | | −.155 |
| .984 | 1.708 | .130 | | | | −.193 |
| .983 | 1.728 | .140 | | | | |
| .986 | 1.697 | .119 | | | | |

addition to the dynamic variations of the state variables of position and velocity, a true dynamic attached to each of the coefficients of the terms entered in the regression. Not only do the coefficients on the linear and cubic Duffing terms vary systematically from cycle to cycle, but so do the coefficients on the Van der Pol and π-mix terms (note that significant Rayleigh terms are mostly absent). Each cycle (and presumably each half cycle) is assembled afresh. An implication of this latter observation is that models will need to be developed that address the observed "parameter dynamics" not only at a coarse-grained level (e.g., Kay et al., 1987, 1991) but also at a fine-grained level. Three possible routes can be taken to modeling these changing coefficients: (a) autonomously, that is, by making the parameters dependent on the system's state variables; (b) nonautonomously, that is, by making the parameters an explicit function of time, and (c) stochastically, that is, by ascribing the observed variations to stochastic forces operating on the parameters. Differentiating among these possibilities is not likely to be straightforward, but a reasonable first step would be to scout for regularities in the observed parameter variations by making them explicitly dependent on time.

The W-method has the special advantage of not only exposing the local prominence of particular terms in the series development of admissible W-terms, but also of revealing the effect of deviations from the circular phase portrait that are too locally constrained to be captured by any of the terms in the series expansion. These "kicks" are discrete in nature and could point to an external control strategy in which a timer closes the perception–action cycle. P. J. Beek and W. J. Beek (1988) and P. J. Beek (1989a) applied the graphic technique to quantifying a typical phase portrait of the hand movement for juggling three balls. The technique revealed the presence of discrete forcing, suggesting that the best dynamical model of its specific phase portrait is one in which discrete, external control action is taken into account in addition to the autonomous nonlinear dynamics of the juggling assembly. The dynamical model implicates a functional braiding of autonomous and nonautonomous control components in the dynamics of juggling. The model equation has the following structure:

$$d^2x/dt^2 + x + f(x, dx/dt)dx/dt + g(x) = A\delta(t = t_C), \qquad (7)$$

where $\delta$ is the unit Dirac function, indicating a pulsed forcing of very short duration and magnitude $A$, at a specific time $t_C$. In theory, it would have also been possible to make the driving term at the right-hand side of Equation 7 dependent solely on the internal states of the system, such as position, velocity, or phase, and thus render it autonomous. In practice, however, this approach was not feasible, because the onset of the kicks could not be linked systemati-

cally (i.e., reproducible from cycle to cycle) to any of the state variables of the hand in motion, nor to the state variables of the other hand in motion.

Applications of the W-function analysis to human rhythmic movements, as in the foregoing, suggest that: (a) the details of simple and more complex rhythmic activities are governed in largest part by autonomous terms (of the W-series); (b) the details not accounted for by (a) are not (entirely) random but seem, at least in some instances, attributable to a nonautonomous forcing function with definite structure; (c) the nature of this forcing function in an instance of rhythmic activity cannot be discerned in the absence of a prior determination of the contributions by the terms of the appropriate W-function, which—in our view—represents control as it is distributed over the entire work cycle, as opposed to the identified discrete control action; (d) this control, however, is not distributed evenly over the entire work cycle, but varies from phase to phase due to the fact that the rate of change of the parameters of the appropriate W-function is faster than a cycle period; (e) the change of these parameters may itself be represented as a nonautonomous dynamic. In the section that follows we suggest a link between the identified discrete nonautonomous forcing functions and information.

## INFORMATION AND COORDINATION

A core feature of biological systems is that information dominates their behavior as much as forces, if not more so. Ecological analysis relates the notion of information, in the context of controlling activity, to lawful regularities (Gibson, 1979). More specifically, it equates informative structures with large-scale kinematic, geometric, and spectral properties of low-energy fields determined lawfully by properties of surface layouts, movements of the body relative to surface layouts, and movements of body segments relative to the body and to each other (Kugler & Turvey, 1987; Turvey, 1990a). Of particular concern to the points we wish to make here is that, with respect to coordinated rhythmic acts, such as 1:1 frequency locking of hand-held pendulums and cascade juggling of three balls, use must be made of information in the foregoing sense, that is, about the macroscopic dynamics of the act.

When a person is asked to swing two hand-held pendulums comfortably at a single, common period (1:1 frequency locking), the experimental evidence suggests that the chosen period corresponds to the minimum of a potential function. The potential function in question is that governing the motion of the simple pendulum equivalent of the two pendulums considered as a compound pendulum (Kugler & Turvey, 1987). We reproduce the function pictorially in Figure 3. As can be seen, there is a trajectory of minimum potential defining the comfort period of absolute coordination. Surrounding that minimum trajectory are trajectories associated with increasingly larger potential values.
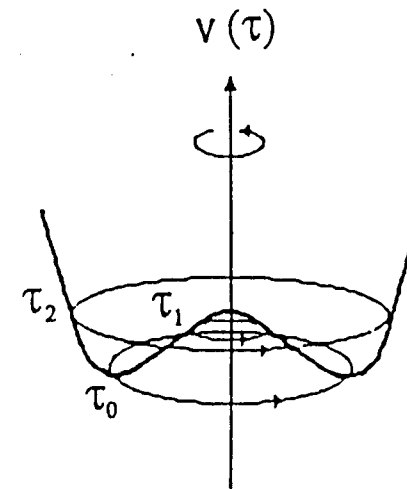
FIGURE 3 Rotating time-dependent potential function $V(\tau)$ for wrist-pendulum systems in 1:1 frequency locking, modeled as a hybrid simple pendulum/mass-spring system. The minimum of the function is defined as the characteristic period $\tau_0$ (after Kugler & Turvey, 1987).

Stable rhythmic movement of two hand-held pendulums in 1:1 frequency locking requires contact with the potential depicted in Figure 3. In understanding the form of this contact, it is important to underscore that the observed rhythmic parameters do not result from the coordinated system being drawn inexorably to the minimum by forces. If the person so chooses, the period of the activity could be at some remove from the period defined by the potential function's minimum. Rather, the observed rhythmic parameters result from the coordinated system being guided intentionally to the minimum by information. In short, there must be information about the potential (Kugler & Turvey, 1987). That is, there must be information about the relevant dynamics of absolutely coordinated units—specifically, their stability conditions. It is on the basis of this information that the aforementioned contact with the potential is achieved. The contact is "soft" (information based) not "hard" (force based). It has been argued by Kugler and Turvey (1987) that this "soft" versus "hard" distinction is crucial to any thoroughgoing dynamical understanding of coordinated rhythmic activity.

In coordinating the rhythmic motions of hand-grasped pendulums, the information of concern is defined over the moving segments of the body. There is a changing pattern of joint articulations and a changing pattern of deformations of cutaneous and subcutaneous tissues brought about by the moving loads, with large-scale consequences for mechanoreceptor activities (Solomon & Turvey, 1988; Solomon, Turvey, & Burton, 1989). In the case of between-person interpendulum coordination, the comfort mode of coordination obeys the same physical principles as in the within-person case, but must now be specified by a combination of haptic information (about the "local" dynamics) and visual information (about the "global" dynamics; R. C. Schmidt, 1991). In

sum, if the goal is to produce reliably 1:1 frequency locking at the most comfortable frequency, then information specifying the potential minimum and deviations from it must be exploited. We hypothesize that exploiting information to achieve and sustain a given trajectory of movement states is indexed by the nonautonomous components in the differential equation(s) representing the activity in question.

Returning to juggling, an argument can be made that this more elaborate activity shares with the simpler one just discussed the characteristic of a potential function defining the act's stability. The argument, unfortunately, is somewhat detailed. We develop it in several graduated steps. To begin with, we note that the "frequency" of the hand movements in smooth juggling must be matched with the "frequency" of the motion of the balls in free fall (P. J. Beek, 1988). A juggling hand loop comprises two distinguishable times: the duration in which the hand carries a ball between a catch and a throw (time loaded, $T_{Li}$), and the duration in which the hand is empty and moving without a ball between a throw and a catch (time unloaded, $T_{Ui}$). A ball loop also comprises two distinguishable times: The duration in which the ball is in a hand (time loaded, $T_{Li}$), and the duration in which the ball is in flight from a hand to the next (flight time, $T_{Fi}$). Now if $N$ is the number of juggled balls, $H$ is the number of hands that do the juggling, $T$ is the duration of a complete $N$-ball, $H$-hand cycle, and $T_L$, $T_U$, and $T_F$ are the average time loaded, time unloaded, and time flight during $T$, respectively, then, from the perspective of a hand, $N(T_L + T_U) = T$, because a hand manipulates $N$ balls during $T$, and, from the perspective of a ball, $H(T_L + T_F) = T$, because a ball is manipulated by $H$ hands during $T$. Hence:

$$N(T_L + T_U) = T = H(T_L + T_F). \tag{8}$$

This timing constraint was first identified by Claude Shannon (Horgan, 1990; Raibert, 1986). It applies beyond the cascade juggle to all recurrent patterns in which a hand never holds more than one ball (e.g., reversed cascade, fountain, shower) and may, therefore, be considered a universal field equation for juggling (Horgan, 1990). Equation 8 can be rewritten to make apparent the manner in which the three times relate:

$$T_F = (N/H - 1)T_L + (N/H)T_U \tag{9}$$

or, given that $H = 2$ and $N = 3$:

$$1 = 1/2(T_L/T_F) + 3/2(T_U/T_F). \tag{10}$$

Let us focus on the ratio of the time a ball is in a hand to the time that that particular ball is in the air ($T_L/T_F$). One can raise the question of the juggler's options with regard to how these two times fill up the loop time of a ball.

Metaphorically, the question is one of how many different ways can the juggler "tile" the total loop time with component times. With respect to fixed, specific areas of two-dimensional space, it is well respected that filling the space completely (so that there are no cracks or gaps) can only be achieved through repetition of a limited number of homomorphic two-dimensional forms (e.g., squares, triangles, hexagons, etc.; Grünbaum & Shephard, 1987; Stevens, 1974). In the notion of phase locking, one finds a similar understanding about filling a given amount of time, that is, there are a limited number of ratios into which component durations can enter. Suppose that these are ratios between relatively prime integers (i.e., integers lacking a common divisor). Then, if the loop time of the ball is "tiled" Pythagorean style:

$$T_L/T_F = 2/1, 3/2, 1/1, 1/2, 1/3, \ldots \tag{11}$$

Given Equation 10, it would follow that

$$T_U/T_F = 0, 1/6, 1/3, 1/2, 5/9, \ldots \tag{12}$$

Looking at the hands, one can imagine that the "temporal tiling problem" reduces, for the juggler, to the question of how long to hold onto a ball during a cycle of the hand. That is, what proportion of the hand's cycle time ($T_L + T_U$) is to be taken up by holding or carrying a ball ($T_L$)? Expressing $T_L$ and $T_U$ in terms of $T_F$ on the basis of the preceding two equations yields:

$$T_L/(T_L + T_U) = 1/1, 9/10, 3/4, 1/2, 3/8, \ldots \tag{13}$$

All of the preceding ratios are permitted by mode-locking considerations. In an experimental analysis of cascade juggling with $H = 2$, $N = 3$, three juggling speeds, and four skilled jugglers, it was found that the duration of the subtask of carrying the juggled object between catch and throw expressed as a proportion of the hand cycle time ($k = T_L/(T_L + T_U)$) ranged between .54 and .83 with mean of .71 (P. J. Beek, 1989a). A regression of $k$ on $T_H = T_L + T_U$ reveals for these data a significant effect of hand cycle time, $k = .21T_H + .57$, $r^2(11) = .42$, $p < .02$; $k$ was larger at lower juggling frequencies. In subsequent experiments, examining juggling with five balls, the proportion of the hand's cycle time taken up by holding or carrying the ball was always close to .75 (range $0.71 - 0.79$; $m = 0.75$). The same proved to be true for three jugglers who could cascade juggle seven balls (P. J. Beek, 1989b; P. J. Beek & Turvey, in press). One reading of the preceding is that there may be a value of $k$ in the vicinity of .75 that jugglers gravitate toward but do not necessarily achieve, and that the degree of attraction towards .75 becomes more severe as $N/H$ increases.

Why should the preferred $k$ be .75 rather than some other candidate rational ratio? During cascade juggling, one ball per hand is maintained at the zenith of

its parabola, from which point the fall time is $T_F/2$. The time-averaged number of airborne balls per hand is given by $N^*/H$ (where $N^* = N - Hk$). If we associate a unit of time $(T_L - T_U)$ with each airborne object per hand, then, by hypothesis, the fall time for the mean number of airborne objects per hand is $(N^*/H)(T_L - T_U)$. A juggler will fill the fall time of objects with $(T_L - T_U)$ time units if $(N^*/H)(T_L - T_U) = T_F/2$. In short, sitting inside the temporal structure of juggling is a convenient principle. It might be referred to as a "tiling principle" insofar as it addresses the question of how the juggler fills up or tiles the loop time of a hand and the loop time of an object with component times (e.g., those of the various repetitive subtasks) so as to ensure a smooth juggle. The principle reads: Tile the loop times of objects and hands in integer multiples of $(T_L - T_U)$ or, to use the simplest possible expression, $P(T_L - T_U) = (T_L + T_U)$, where $P$ is an integer. In combination with $k = T_L/(T_L + T_U)$, it follows from this that $k = 1/2 + 1/(2P)$, with $P > 1$ and $1/2 < k < 1$. Self-evidently, $P$ cannot equal 1, but it can assume any real value greater than 1. The assumption that principles of frequency locking apply suggests that integer values of $P$ will be associated with stable regimes and that the lowest possible integer at which frequency locking can occur, namely 2, will be associated with the most stable regimes. At integer values of $P > 2$, frequency locking will occur at $k$ values less than $3/4$ (.75); for example, for $P = 3$, $k = 2/3$, for $P = 4$, $k = 5/8$, etc., as indeed was found in the case of juggling three balls (P. J. Beek & Turvey, in press).

In sum, the empirical evidence suggests that cascade juggling is describable by a potential function with multiple minima corresponding to the integer values of $P$ at which mode locking can be achieved (P. J. Beek, 1989b; P. J. Beek & Turvey, in press). The number of minima is a function of the number of balls being juggled, and appears to reduce to one ($k = 3/4$) in the case of juggling five balls and more. That is, the juggler performing cascade juggling, like the person coordinating two hand-held pendulums, organizes his or her movements with respect to the minima of a task-dependent potential function. We presume that information about the function, its minima and its gradients, is available to the juggler. And we presume further that the exploitation of this information is revealed in the nonautonomous (right-hand) terms of Equation 7.

## INFORMATION SECURES AUTONOMY

Let us pursue the juggling case a little further to underline an important aspect of the informational character of human movement. As noted, the nonautonomy of Equation 7 is expressed in terms of a Dirac delta function (a sharp forcing pulse that operates each time its criterion is satisfied). This fact raises particular information-related questions such as, what is the minimal number and size of the kicks needed to sustain the desired trajectory? The

informative structures available to the juggler are defined over (a) covariations in tissue deformations and branchings of the bones (cf. Gibson, 1966; Solomon & Turvey, 1988) and (b) optical structure, particularly that generated by the flights of the balls and their phasing (P. J. Beek, 1988). Considerations from the ecological point of view (e.g., Fitch & Turvey, 1978; Kugler & Turvey, 1987) would suggest that, in assembling and executing the act of juggling, a functional specification must be defined over the informative structures and the identified nonautonomous components such that informational properties specify forcing properties (e.g., when to force and how much to force). This way of thinking is in concert with the thesis that any coordinated activity is simultaneously an organization of outflow ("efference") and an organization of inflow ("afference"; Gelfand et al., 1971; Turvey, 1990b), that is a closed perception–action cycle.

The functional linkage of information and forcing suggests how it is that coordinated human movements may submit to completely autonomous descriptions even though nonautonomous terms might figure prominently in their analyses. We have seen how the description of the motions of the body segments in juggling must entail both autonomous and nonautonomous dynamics for completion. The lesson of the analyses described in the present article is that, when restricted to the level defined by the motions themselves, a coordinated act need not be an autonomous dynamical system. The promise of the preceding paragraph, however, is that when analysis is raised to the level that incorporates the perception–action cycle, a coordinated act is an autonomous dynamical system. If the forcing function is information based, as argued, then the clock time that appears explicitly in the formulation of the forcing function must be interpreted as a placeholder. It is a placeholder for an information variable. The proposed functional linkage between information and forcing is tantamount to a claim that forcing is a function of information, not clock time. The proposal suggests that after a (nonautonomous) forcing function has been determined and formulated precisely for a given instance of coordinated rhythmic activity, the next step is to determine and formulate precisely the activity's informational basis. The goal is to identify the information variable or variables that can replace time with the results of (a) satisfying fully the dynamics of the act and (b) rendering the overall dynamical description autonomous. An educated guess is that the relevant information variable governing the right-hand side of the cascade juggling equation, Equation 7, may be a generalization of the time-to-contact optical quantity investigated most notably by Lee (1980; P. J. Beek, 1988, 1989b; see also, Bootsma & van Wieringen, 1990).

We can now get reasonably clear on the way in which the dynamical description of the juggling movements provided by Equation 7 is to be interpreted. At first blush, the differences between the two kinds of dynamics evident in Equation 7, the autonomous and the external forcing function, suggest that they might map onto distinctions of a more familiar nature, for example, peripheral and central nervous systems, spinal and cortical motor components,

or nonintentional and intentional processes. To construct such mappings, however, would give a misleading picture of juggling's dynamical basis.

The familiar distinctions in question reduce ordinarily to a dominance or superimposition relation. One system, component, or process dominates, or is superimposed upon, the other. If such distinctions did bear on the contrast of autonomous and nonautonomous dynamics in coordinated rhythmic movements, then we would have to suppose that the two dynamics relate similarly by way of dominance or superimposition. For example, it could be hypothesized that the basic dynamic of juggling is captured by the autonomous components, with the nonautonomous components comprising an independent source of adjustment that is added onto the autonomous components. By this hypothesis, the two kinds of dynamics represented in Equation 7 are separable physically, and not just logically or mathematically.

An alternative hypothesis—one that follows from the preceding discussion of a functional linkage of information and forcing—is that the two kinds of dynamics represented in Equation 7 are not related through dominance or superimposition but rather are dually complementing, such that Equation 7 represents a truly unitary dynamical regime. The first hypothesis conveys the image of the juggler engaging in two distinct processes, namely, assembling the autonomous components of juggling and then assembling, as an overlay, the forcing function, albeit not in the suggested serial order. The alternative hypothesis conveys the image of the juggler engaging in one process, namely, assembling a single dynamical regime comprising (a) nonlinear (W-)terms and (b) a forcing function that exploits the information generated lawfully by the autonomous dynamics, where (a) and (b) are being treated, at least temporarily, as interdependent, but functionally distinct, components. The significant implication of the alternative hypothesis is that the dynamics of a skill like juggling are composed intrinsically of autonomous components and forcing, and that this might apply generally to all coordinated rhythmic movements.

The foregoing interpretation of the autonomous/forcing relation bears on the understanding of movement's intentionality. In colloquial terms, an intention is equated with doing something according to will or choice, and is distinguished from doing something reflexively or instinctively. The decisive idea behind the colloquial usage is that of personally bringing about a change in an ongoing state of affairs that is otherwise proceeding under its own devices. For this reason it is easy to see how one might wish to view the kicks that characterize the nonautonomous components of the juggling dynamic as being particularly good expressions of the intentional part of that activity. Be that as it may, the alternative hypothesis just mentioned would suggest that the kicks are no more or less an expression of intentionality than the continuous, nontransitory aspects of the juggling dynamic as represented by Equation 7.

In technical philosophical terms, intentionality refers to the fact that mental states exhibit an "aboutness" or a "directedness." They are states that are about

objects, objectives, significances, or meanings or, synonymously, states that are directed toward such things. Ordinary sentences express intentionality through the use of an intentional or psychological idiom, such as "I wish to do x," where "wish" is an example of an intentional idiom and "to do x" is an example of what might be termed intentional content. It helps to paraphrase sentences of this kind as follows: "I wish that I do x," because the paraphrase brings out the fact that the auxiliary "that" connects all intentional or psychological idioms with their corresponding intentional contents. One consequence of the technical treatment of intentionality is that it leads to a view of an activity such as swinging two hand-held pendulums in a 1:1 frequency pattern, or an activity such as juggling, as infused in its entirety by intentionality. Thus, for example, the intentional state expressed roughly as "I want that I keep these three balls moving between the two hands continuously" can only be satisfied by an organization that incorporates both the forcing function and the autonomous components identified in Equation 7. Issues can be broached, therefore, of how to proceed toward an understanding of the actual intentional contents of 1:1 interlimb frequency locking and juggling given knowledge of the form of the macroscopic dynamics that characterizes these acts.

We suggest that answers to the preceding are shaped by the fact that 1:1 frequency locking of differently sized hand-held pendulums, and the cascade juggling of three balls, involve physical laws and principles that have to be tailored in a particular way in order for each to take place. As a first approximation, therefore, it must be the case that intentionality *harnesses* dynamics, and the informative structures that they make available, in a highly particular manner (cf. Kugler & Turvey, 1987; Turvey, 1986, 1990a). As such, intentional content must play a role analogous to the auxiliary conditions (initial conditions, boundary conditions, constraints, etc.) of classical mechanics. More directly, intentional content must comprise such conditions with respect to the physics and information pickup that makes coordinated movements possible. In regard to information pickup, intentional content would seem to entail a classification of regions of the coordination pattern as those to which attention should be directed and within which forcings should be applied.

## INFORMATION SECURES A BALANCE BETWEEN STABILITY AND ADAPTABILITY

It remains to identify a special property that belongs to systems that are governed more by information than by forces. As noted, the juggling data reveal that jugglers do not occupy the regions of mode locking but that they function at the borders of these regions, with the better jugglers functioning closer to those borders than the poorer jugglers. A system that functions habitually inside a mode-locked region is a system for which the initial conditions are repeatable

exactly from one instance of functioning to the next. For an ordinary biological movement system the requirement of reproducible initial conditions cannot be satisfied. Biological movement systems assemble the macroscopic parameters of oscillation afresh each cycle, and do so from a microstructure of very many degrees of freedom. They are also prohibited from residing in the mode locking regions by virtue of their informational basis. As noted, a region of mode locking is a region associated with a potential minimum. By the arguments advanced here, a system that relates to a potential function through information can hover about the vicinity of the potential minimum to the degree that (a) it can register the information specifying the minimum and the magnitude of departures from the minimum, and (b) its forcing terms are linked to the information. This latter fact, expressing the general understanding that the juggler is not forced into a minimum but must discover it through information, means that the juggling assembly functions habitually outside, but close to, the borders of mode-locking regimes. There is a distinct benefit to this way of functioning, namely, juggling can be stable but at the same time adjusted relatively freely to accommodate perturbations in the routine. Adaptability to varying circumstances, therefore, is a simple but highly significant consequence of the fact that coordinated movements are dynamical systems governed by information.

We can reinforce the preceding ideas by a final consideration of a subject attempting to swing two pendulums comfortably out of phase at the same frequency. Figure 4 shows, for three subjects (R. C. Schmidt et al., 1991), the time series of the continuous phase relation between the two units under three conditions, one in which the two pendulums are nearly identical (right/left ratio of uncoupled eigenfrequencies, omega, is nearly unity) and two in which the two pendulums are different ($\Omega = 1.91$ and $\Omega = .52$). The main features to be noted about Figure 4 are: (a) in all omega conditions the mean frequency of the right unit and the mean frequency of the left unit were almost identical, that is, absolute coordination was achieved in the average state; (b) in all omega conditions there was considerable moment-to-moment variation in phase, that is, the moment-to-moment interlimb pattern was primarily that of relative coordination (meaning that neither frequency nor phase locking was achieved); (c) the average deviation from the intended phase relation was larger for $\Omega = 1.91$ and $\Omega = .52$ than for $\Omega = 1.03$; (d) in all omega conditions there were tendencies toward phase locking at 180°; and, relatedly, (e) in all omega conditions there are hints that satisfying the phase demands of the task may be limited to particular phase values. These features of the interlimb coordination of pendular rhythmic movements reinforce the impression of imperfect mode locking, that is, of an attraction to modes (well-defined frequency and phase relations) without a locking into modes—the coordination is relative with mode attraction, not absolute with mode locking. In paraphrase of the conclusion drawn from juggling, the strategy of gravitating toward mode-locking regimes but operating on their edges gives to coordination patterns an essential balance
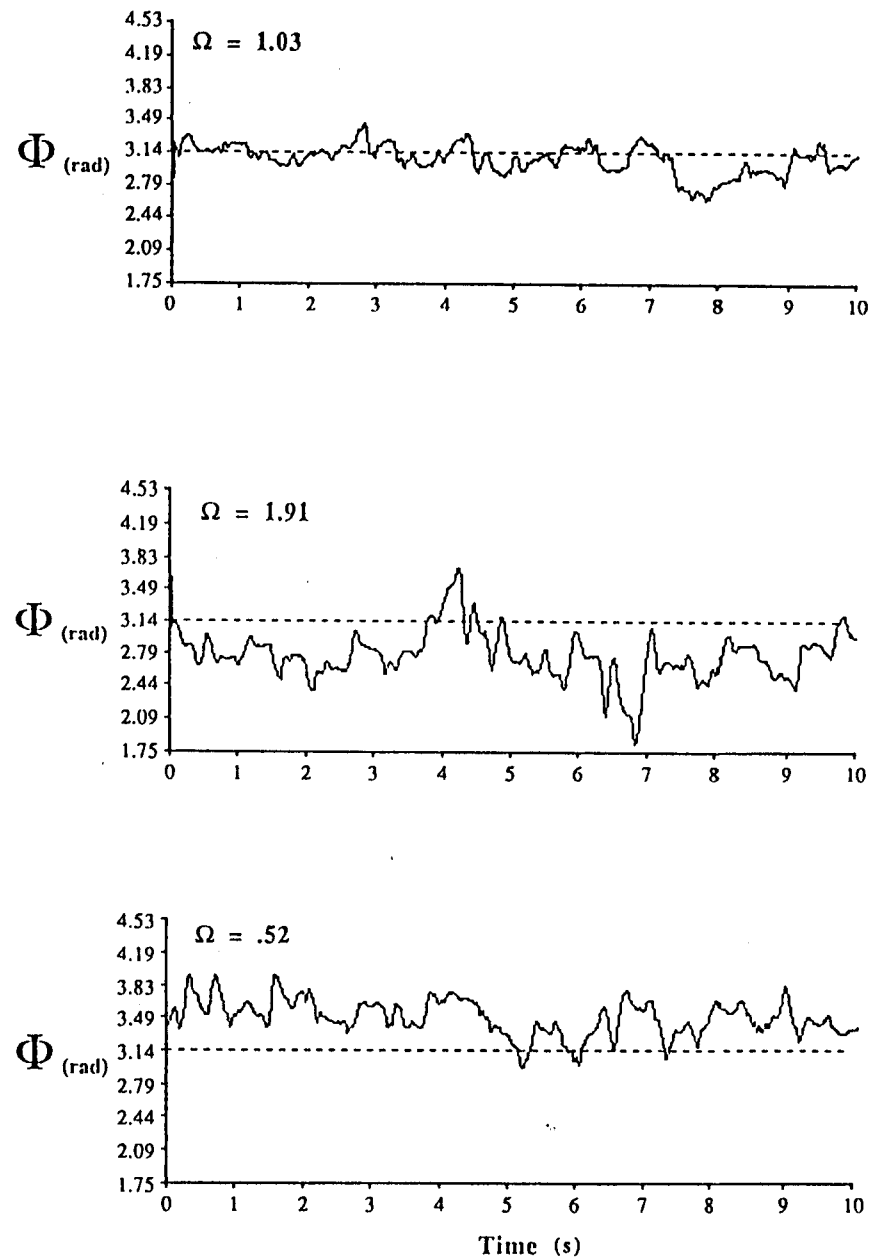


FIGURE 4  Time series for the relative phase $\Phi$ of two rhythmic units (wrist-pendulum systems) achieving 1:1 frequency locking in the mean, as a function of the ratio of the uncoupled frequencies $\Omega$.

between persistence and change (P. J. Beek, 1989a, 1989b; Turvey, 1990b; Turvey & P. J. Beek, 1990; Turvey, R. C. Schmidt, & Beek, in press).

## ADDITIONAL ADVANTAGES OF INCORPORATING NONAUTONOMOUS DYNAMICS

We conclude with an overview of what we see as other benefits of a modeling strategy that does not a priori exclude—and, when possible, attempts to exploit—the incorporation of nonautonomous components in the description of biological movements.

### Challenge of System Decomposition

It is true, of course, that the laws of nature do not vary with time. Nonetheless, equations with a time-dependent right-hand side, Equations 2 and 7, for example, will arise in situations where the system component under study is part of a compound system, and when this component is influenced unidirectionally by other components. In this case, the other "forcing" components evolve independently of the first component, and the state variables of these forcing components can be used to define a clock-time dependent forcing function for the now nonautonomous dynamics of the first component. Suppose we consider some part P of a physical system P + Q. Although it is the case that the law of evolution of the whole system does not vary with time, the influence of part Q on part P may cause the law of evolution of part P to be time dependent. For example, the influence of the moon on the earth produces tides, and this influence is expressed mathematically by the fact that the magnitude of the acceleration due to gravity, which figures in the equation of motion of terrestrial objects, becomes time variable, and that the rest of the system has no effect on it. Similarly, the observed behavior of an action system can be expected to reflect multiple subsystems, multiple subsystems that need not always be coupled bidirectionally. If this is the case, then the most effective procedure is to construct first the dynamics of part P of the system and to then represent the net action of all other parts of the system (Q, R, etc.) as one, time-dependent, forcing function. Moreover, if the subsystems of movement systems are related bidirectionally, it may be useful to allow for nonautonomous dynamics during some stages of the modeling process. By first modeling the effect of one subsystem on another subsystem as a nonautonomous dynamic, and vice versa, one can attempt to discover, by comparing model predictions with actual data, what type of bidirectional (autonomous) coupling term should be designed to construct a more adequate model. Allowing for, and capitalizing on, nonautonomous dynamical descriptions in addition to autonomous dynamical descriptions promotes the identification of effective system decompositions.

### Challenge of Identifying Neural Constraints

A natural-physical approach to human movement aims at understanding the formative physical strategies at work in human and animal movements in such a way that the role of neural subsystems will become apparent (Turvey, 1990b). Pursuing a nonarbitrary distinction between autonomous and nonautonomous components to reveal the dynamics that underwrite biological movements enhances the likelihood that a functional rather than a morphological identification of neural subsystems may be reached (i.e., in terms of physically intensive variables and their uni- or bidirectional coupling). Ultimately, a major aim is a general understanding, in general dynamical terms, of the interactional organization among individual subsystems at the level of the central nervous system (Schöner & Kelso, 1988). Nonarbitrary inclusion of nonautonomous components would aid this effort, arbitrary inclusion would hinder it.

### Challenge of the Heterogeneity of Composition

Most dynamical analyses are with respect to systems that involve enormously large degrees of freedom that are of like kind (e.g., the photons present in a neon tube or the electrons in electrical circuits). Biological action systems, however, are more properly interpreted as heterogeneous rather than homogeneous complex systems, consisting, as they do, of various morphological structures that involve various functional processes. In contemporary physical language, movement systems are probably systems that are to a considerable extent "frustrated" (Chowdhury, 1986), meaning that their varied components at multiple scales are subject simultaneously to very many different physical requirements that they cannot possibly satisfy fully. To the extent that their subsystems are related unidirectionally, the dynamics of heterogeneous systems may necessitate the inclusion of nonautonomous components.

### Challenge of Defining the Relevant Physics

The understanding of coordination in law-based terms is in its infancy. Consequently, it is premature to restrict the potent conceptual framework of nonlinear dissipative dynamics exclusively to those concepts that are designed to capture autonomous processes of spontaneous pattern formation (self-organization). Such a restriction may have the operational advantage of narrowing the focus in such ways that phenomena of self-organization in human movement are addressed from the perspective of the theory of autonomous systems alone. This is not to say, however, that including nonautonomous dynamical ingredients would necessarily render modeling procedures underconstrained and explanations more arbitrary. As a case in point, the behavior exhibited by (either

continuously or discontinuously) driven oscillators can be modeled as rigorously as the behavior of completely autonomous systems.

The theory of an important class of physical phenomena, such as Farey phase locking (e.g., Chernikov, Sagdeev, Usikov, Yu Zakharov, & Zaslasvsky, 1987), parametric resonance (Arnol'd tongues, Arnol'd [1983]), and frequency-, phase- and amplitude modulation due to "messenger signals," is (at least) equally well developed in the context of externally driven, nonautonomous systems as it is in the context of autonomous systems. The entire menagerie of dynamical patterns can be generated in systems with (appropriately parameterized) autonomous or nonautonomous dynamics of at least third order, but they may possess nontrivial dynamical differences. Making the assumption that coordinative structures may involve both autonomous and nonautonomous dynamical components means that the mathematical richness of the framework of nonlinear dynamics can be exploited to the fullest in making the informational basis of movement transparent. The booming field of nonlinear dynamics (e.g., Thompson & Stewart, 1986) offers a rich conceptual reservoir and a heterogenous testing ground to not only identify the type of dynamics comprising human action systems, but also, we trust, to come to terms with the informational nature of these systems. Necessarily, the first step in developing a physics suitable to an understanding of the problem of coordinated movement is one of identification. As Iberall (personal communication) would put it, the central question is, what is the physics of this problem?

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