Diversity and commonality in music performance: An analysis of timing microstructure in Schumann's "Träumerei"

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This study attempts to characterize the temporal commonalities and differences among distinguished pianists' interpretations of a well-known piece, Robert Schumann's "Träumerei." Intertone onset intervals (IOIs) were measured in 28 recorded performances. These data were subjected to a variety of statistical analyses, including principal components analysis of longer stretches of music and curve fitting to series of IOIs within brief melodic gestures. Global timing patterns reflected the hierarchical grouping structure of the composition, with pronounced *ritardandi* at the ends of major sections and frequent expressive lengthening of accented tones within melodic gestures. Analysis of local timing patterns, particularly of within-gesture *ritardandi*, revealed that they often followed a parabolic timing function. The major variation in these patterns can be modeled by families of parabolas with a single degree of freedom. The grouping structure, which prescribes the location of major tempo changes, and the parabolic timing function, which represents a natural manner of executing such changes, seem to be the two major constraints under which pianists are operating. Within these constraints, there is room for much individual variation, and there are always exceptions to the rules. The striking individuality of two legendary pianists, Alfred Cortot and Vladimir Horowitz, is objectively demonstrated here, as is the relative eccentricity of several other artists.

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INTRODUCTION

A. Diversity and commonality in music performance

More than at any earlier period in musical history, the contemporary scene in serious music is dominated by the performer (see Lipman, 1990). Music consumers thrive on a limited repertoire of standard masterworks, primarily from the 19th century, that are offered again and again in different performances, both in live concerts and on recordings. The great musical events of our time are not the premieres of new compositions, but the appearances and reappearances of superstar conductors and instrumentalists. Young musicians compete for career opportunities by entering competitions in which their performances are compared and evaluated by juries (see Cline, 1985; Horowitz, 1990). While ever new renditions of the standard repertoire vie for the attention of record buyers and concert audiences, a spate of reissues of historical recordings on CDs is offering stiff competition. For some of the more popular works, the Schwann catalog lists dozens of performances; if deleted records, available in libraries and private collections, are counted, they may run into the hundreds. There have never been such ample opportunities to compare different performances of the same music.

Obviously, this remarkable diversity reflects not only clever marketing and the promotion of superstars, but also the ability of music lovers to distinguish and appreciate different performances. The more sophisticated among these listeners detect qualities in the performances of individual artists that lead them to search out these performers' concerts and recordings. They may voice opinions about the quality of particular performances by these and other artists, describing them as "brilliant" or "noble" or "thoughtful." Some of the more gifted professional critics excel in characterizing different performances in terms alternatingly scholarly and poetic.

While the attention of listeners and critics thus is mainly drawn to the *differences* among performances, there are also strong *commonalities*, usually taken for granted and hence unnoticed. Although there is a large variety of acceptable performances of a given piece of music, there is an even larger variety of unacceptable performances, which rarely make their way into the concert halls or onto records. Unless they have the mark of inspired iconoclasm (as do some of the performances by the late Glenn Gould; see, e.g., Lipman, 1984), they quickly succumb to the fierce competition of the musical marketplace. Music teachers, however, have to deal with them every day and try their best to mold immature and wayward students into performers that can be listened to with pleasure. Although teachers may differ considerably in their methods and goals, and are rarely very explicit about what these are, they are transmitting the unwritten rules (though see Lussy, 1882) of a performance tradition that goes back to 19th century central Europe, where most of the standard repertoire originated. Despite various changes in performance practices during the last 200 years, most of them of a narrowly technical nature, there are generally accepted *norms* of musical performance, according to which the artist's actions are largely subordinated to the musical structure. The artist's primary task is the *expression* of the musical structure, so it can be grasped and appreciated by the listener, and make an *impression* on him or her. (See

Lussy, 1882; Riemann, 1884; Stein, 1962.) This is presumably done by conventional means that are adapted to the hearers' perceptual and cognitive abilities. However, the particular doses in which these techniques are applied (unconsciously, for the most part) vary from artist to artist and account for individual differences in interpretation.

Thus there are two basic aspects of music performance: a *normative* aspect (i.e., commonality) that represents what is expected from a competent performer and is largely shared by different artists, and an *individual* aspect (i.e., diversity) that differentiates performers. The individual aspect may be conceived of as deviations from a single ideal norm; more profitably, however, it may be thought of as individual settings of free parameters in the definition of the normative behavior. That way, it is possible for different artists to meet the norm equally and yet be discriminably different. For example, two pianists may be equally adept in expressing a particular musical structure in their performances, but one may choose a slow tempo and large tempo changes, whereas the other may prefer a faster tempo and smaller deviations from the rhythmic beat. The former artists may then be characterized as "romantic" or exuberant," while the latter will evoke epithets such as "restrained" and "noble," though both may receive equal acclaim from a sensitive audience.

The commonality–diversity distinction seems obvious enough, but there is little tangible evidence to substantiate it. Although volumes have been written about different performers and their characteristics, these discussions rarely go beyond generalities, and the vocabulary used (such as the adjectives quoted above) are not specifically linked to particular performance properties; they also may be used differently by different writers. There is no consistent terminology, nor a well-developed and generally accepted theory of performance description and evaluation, that would make possible an objective characterization of performance commonalities and differences. Music criticism is an art rather than a science, and the critic's impressions, accurate as they may be, are filtered through an idiosyncratic web of personal experiences, expectations, preferences, and semantic associations. In fact, the belief is widespread that performance differences *cannot* be characterized objectively.

Such a negative conclusion, however, can only be justified if objective performance analysis has been attempted and has failed. A total failure is highly unlikely, however, for there are many physical properties of a musical performance that, without any question, *can* be measured objectively. Whether objective analysis can capture a performance *exhaustively* may remain in doubt; the proper question is how much can be learned from it. Surely, even a partial characterization in terms of verifiable and replicable observations can make a valuable contribution to our understanding of performance commonalities and differences, which remain mostly a mystery to this day. For example, most music lovers would agree that Artur Rubinstein and Vladimir Horowitz were two very different pianists, whose performances of similar repertoire (e.g., Chopin) are instantly distinguishable. But what is it, really, that makes them so different and individual? And do they have anything in common at all? It is all too easy to couch the answers to such questions in terms of "artistic personality," which do not enlighten us at all about the nature of the differences and commonalities. Objective performance analysis has a contribution to make here.

B. Objective performance analysis

The concept of objective performance analysis goes back to Seashore (1936, 1938, 1947) and his collaborators, who pioneered the use of acoustic analysis techniques to derive "performance scores" that show the exact variations of pitch, timing, and intensity produced by an artist on some instrument. A considerable amount of data was collected in Seashore's laboratory, but their analyses remained rudimentary and focused primarily on technical aspects such as pitch accuracy and *vibrato* in singing and string playing, and chord synchrony on the piano. Although some studies compared different performances of the same music, no statistical characterization of commonalities and differences was attempted, nor were the results interpreted with more than a passing reference to musical structure. These limitations reflect the behavioristic approach of American psychology at the time, as well as the unavailability of psychological performance models and advanced statistical methods. Nevertheless, Seashore (1947, p. 77) was able to reach conclusions that reinforce the premises of the present study:

> "...there is a common stock of principles which competent artists tend to observe;... We should not, of course, assume that there is only one way of phrasing a given selection, but, even with such freedom, two artists will reveal many common principles of artistic deviation. Furthermore, insofar as there are consistent differences in their phrasing, these differences may reveal elements of musical individuality."

Contemporary with Seashore's work, similar research was going on in Germany. Hartmann (1932) compared the timing patterns of two famous pianists' performances of a Beethoven sonata movement and provided detailed numerical descriptions of the differences between them. The small size of the sample, however, limits the generality of his conclusions. Although one of the two artists seemed quite eccentric, this impression cannot be substantiated without more extensive comparisons.

The three decades between roughly 1940 and 1970 were barren years for the objective study of music performance. In the last two decades, however, several researchers have taken up the topic again. Their work, with few exceptions, has taken a case study approach; their focus was not on individual differences but on the instantiation of certain principles in (hopefully, representative) performance samples, mostly from pianists. Thus Shaffer (1980, 1981, 1984, 1989; Shaffer *et al.*, 1985) examined the timing of piano performances from the perspective of motor programming and control (see, also, Povel, 1977). His erstwhile students, Eric Clarke and Neil Todd, went on to make significant contributions of their own. Clarke (1982, 1985) conducted case studies of piano performances of Satie's music and wrote at length on how musical structures are expressed in timing variations

(Clarke, 1983, 1988). Todd (1985, 1989; Shaffer and Todd, 1987) developed a computational model of timing at the phrase level, which has been revised and extended to dynamics in his most recent work (Todd, 1992a,b). Bengtsson and Gabrielsson (1980; Gabrielsson, 1974; Gabrielsson *et al.*, 1983) conducted extensive studies of the performance of different rhythm patterns in relatively simple musical contexts, but with attention to individual differences. Gabrielsson (1985, 1988) has written more generally about timing in music performance. A provocative theory of "composer's pulse" in performance timing has been developed by Clynes (1983, 1987).

There are few studies in the literature that employed what one might consider a representative sample of different performances. Gabrielsson (1987) conducted a detailed comparison of five pianists' performances of the first eight measures of Mozart's Piano Sonata in A major, K. 331; the study included measurements of timing, intensities, and articulation (i.e., tone durations). Palmer (1989) studied the same music as performed by six pianists in "musical" and "unmusical" styles, and she analyzed the timing patterns, note asynchronies, and articulation. Palmer also recorded eight pianists playing the first 16 measures of a Brahms Intermezzo and studied timing patterns in relation to intended phrasing. She concluded that "pianists share a common set of expressive timing methods for translating musical intentions into sounded performance" (p. 345). These studies still used relatively small samples and obtained only limited amounts of data from each performer.

The largest amount of performance data was analyzed in Repp's (1990) study, which included 19 complete performances by famous pianists of a Beethoven sonata movement. The analysis was limited, however, in that it concerned primarily timing patterns at the level of quarter-note beats. Also, the focus of the study was a search for Clynes' (1983) elusive "Beethoven pulse" in the timing patterns. Following the example of Bengtsson and Gabrielsson (1980), Repp applied principal components (factor) analysis to these timing data, to determine how many independent timing patterns were instantiated in this sizeable sample of expert performances. Two factors emerged, the first representing primarily phrase-final lengthening, while the second factor captured other types of expressive timing variation. Musical listeners' evaluations of the performances were also obtained, and some relationships between the measured timing patterns and listeners' judgments were found.

The present study continued the general approach taken by Repp (1990), but without the aim of testing Clynes's theory of composer's pulse. Its main purpose was to assemble a large sample of performances of a particular composition by outstanding artists, and to analyze the timing patterns in detail, using various statistical methods. These methods, it was hoped, would make it possible to separate commonalities from differences. The common patterns would reveal how most pianists transmit musical structure and expression through timing variations, and it was of interest to determine whether there would be a single common factor or several. The characterization of individual differences was also of interest, especially with respect to some of the legendary pianists in the sample, who are known for their individuality.

The music chosen for this investigation was a well-known piano piece from the Romantic period, "Träumerei" by Robert Schumann. It was selected because it is a highly expressive piece that permits much freedom in performance parameters, and hence much room for individual differences in interpretation. Also, there are numerous recordings available.

I. THE MUSIC

"Träumerei" ("Reverie," "Dreaming") is the seventh of the 13 short pieces that constitute Robert Schumann's (1810–1856) "Kinderszenen" ("Scenes from Childhood"), op. 15. This little suite, universally considered one of the masterpieces in its genre, was composed by Schumann in 1838 when he was secretly engaged to Clara Wieck. The pieces were selected from some 30 pieces composed for Clara around that time, and their titles may have been added as an afterthought. They are not intended for children but rather reflect an adult's recollection of childhood (Brendel, 1981; Chissell, 1987).

"Träumerei" occupies a central position in the "Kinderszenen" suite, not only by its location but by its duration and structural role. It serves as a resting and turning point in the cycle, which shows so many intricate thematic connections that it may be considered a set of free variations (Reti, 1951; Traub, 1981). Its key signature, meter, and motivic content also single it out as the hub of the suite (Brendel, 1981; Traub, 1981). However, it is also often performed by itself, both by classical artists (e.g., it was one of Horowitz's favorite encores) and in numerous popular versions and arrangements. Indeed, "Träumerei" is perhaps *the* most popular Romantic piano piece. Its score is shown in Fig. 1.

The melodic/rhythmic structure (or grouping structure; see Lerdahl and Jackendoff, 1983) of "Träumerei" is depicted schematically in Fig. 2, which also introduces terminology to be used throughout this paper. The layout of the figure corresponds to that of the score in Fig. 1, and bars (measures) are numbered. The piece is composed of three 8-bar periods (A, B, A′), the first of which is obligatorily repeated. Each period is subdivided into two 4-bar phrases, which are represented by staff systems in Fig. 1 and by large rectangular boxes in Fig. 2. (Actually, the beginning and end of each phrase extend slightly beyond the four bars, overlapping with the preceding and following phrases, respectively.) There are two phrase types, a and b, each of which recurs three times with slight variations (indicated by subscripts in Fig. 2). Phrase a1 (period A) is repeated literally in period A′. Phrases b2 and b3, which constitute period B, are structurally identical but differ in key, harmony, and some other details.

The melodic events are divided horizontally (roughly, along the dimension of relative pitch) among four registers or voices (S = soprano, A = alto, T = tenor, B = bass). Vertically (along the dimension of metrical distance), the events within phrases are grouped into *melodic gestures*, which are represented by filled boxes in Fig. 2. Characteristi-

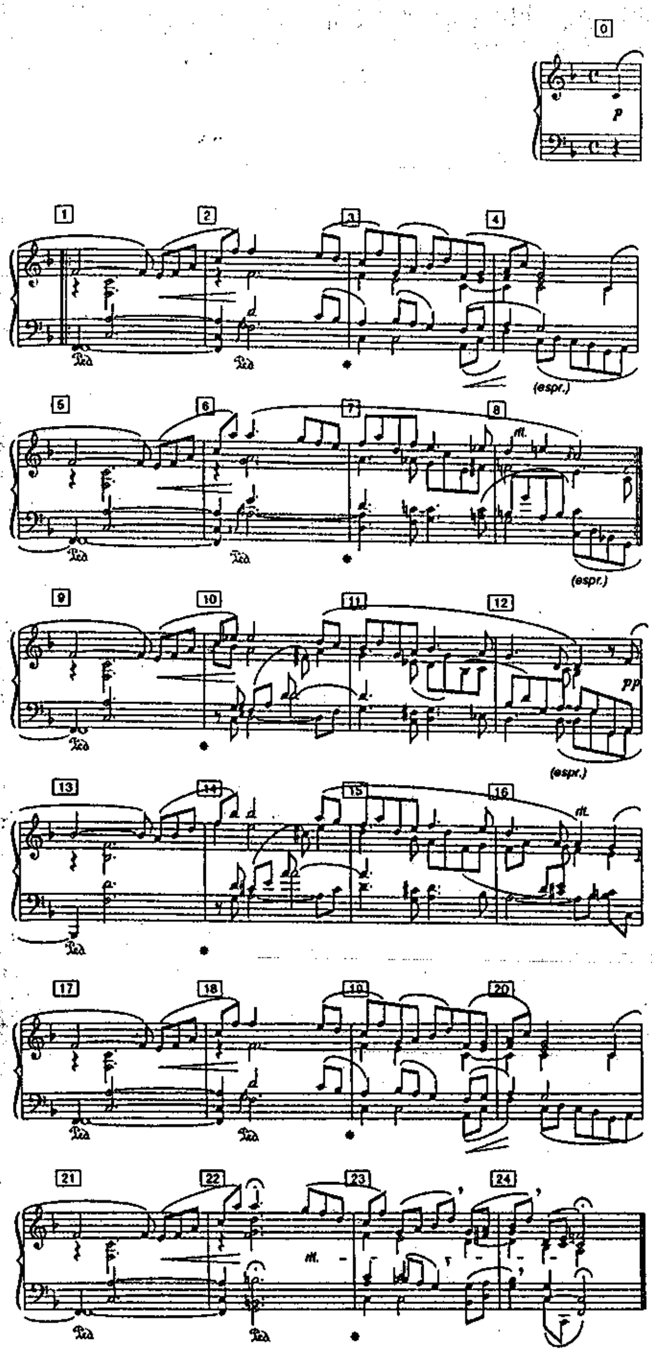

FIG. 1. The piano score of "Träumerei," created with MusicProse software following the Clara Schumann (Breitkopf and Härtel) edition (with some deviations in minor details due to software limitations). The layout of the score on the page is intended to highlight the structure of the music.

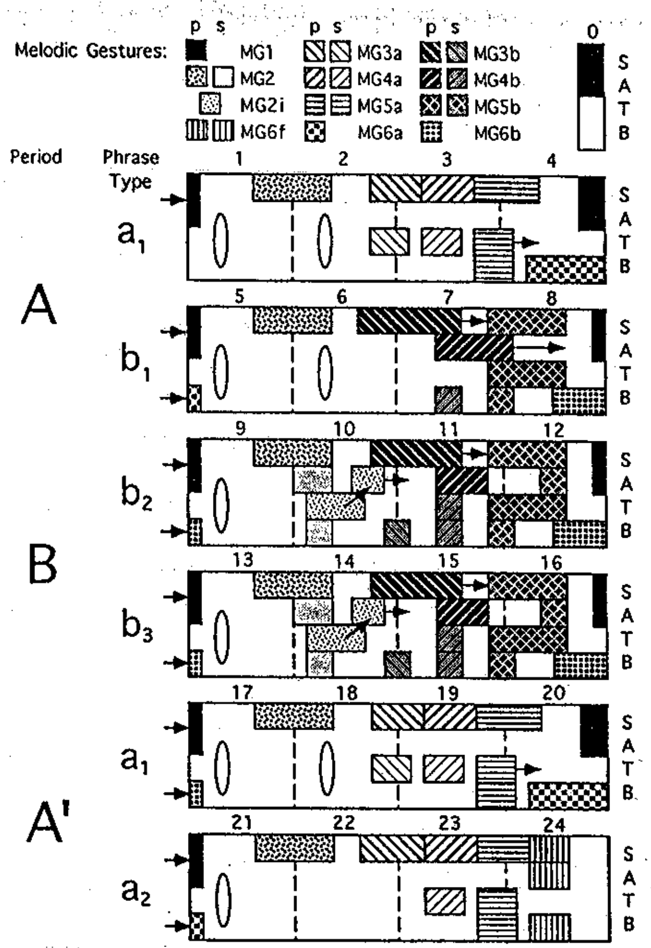

FIG. 2. Schematic representation of the melodic/rhythmic structure of "Träumerei." (See text for explanation.)

cally, they extend across bar lines (vertical solid and dashed lines in Fig. 2). A melodic gesture (MG) is an expressive unit composed of at least two and rarely more than seven successive tones. It is defined here to begin with the onset of its first tone and to end with the onset of its last tone. (In Fig. 2, each MG box extends one eighth-note space beyond the metrical onset of the last tone.) Blank spaces represent time spans devoid of MGs; they may contain single tones, sustained tones, or rests. Multivoiced chords having some gestural quality are represented by vertical ellipses in Fig. 2. Arrows indicate continuity of a MG across a line break or with a subsequent melodic event. Thus MG1 and MG6 continue from the end of one line to the beginning of the next, and MG3b and MG5b in the soprano voice are closely linked. When an arrow points into blank space, it points to the onset of a single-tone event that coheres with the MG.

MGs are divided into primary (p) and secondary (s) ones. The latter are usually shorter and accompany primary MGs; they are represented by boxes filled in lighter shades. An exception to this classification is MG2i, a delayed imitation of MG2 divided between the tenor and alto voices (bars 10 and 14). In the following timing analyses, we will essentially be concerned only with the primary MGs, which represent the leading voice(s) in the polyphonic quartet. As can be seen in Fig. 2, in phrases of type a the primary MGs (including the final MG6f in bar 24) are all in the soprano, except for MG6a which is in the bass and overlaps both MG5a and MG1. In type b phrases, during their second half, the primary MGs cascade down through the four voices, overlapping each other.

II. THE PERFORMANCES

The sample of performances analyzed here includes 28 performances by 24 outstanding pianists, selected according to ready availability (see Table I). Two artists, Cortot and Horowitz, are represented by three different recordings each. Most of the recordings (17) are on long-playing records, 3 are on cassettes, and 8 are on CDs (including a transfer of a 1929 recording by Fanny Davies, a one-time student of Clara Schumann, from a very scratchy original). Most of the performances are of the complete "Kinderszenen," but five are of "Träumerei" only. Two of the latter are from live concerts; all others are studio recordings. Table I includes actual or estimated recording dates.

The 24 artists include some of the most renowned pianists of this century as well as some less well-known artists. They can be grouped according to gender (6 female, 18 male), country of origin (5 from Russia; 4 each from Austria and France; 3 from England; 2 each from Germany and Brazil; one each from Czechoslovakia, Argentina, and Chile; one unknown), and approximate age at the time of recording (about equal numbers of young, middle-aged, and old). At least 12 pianists are no longer alive.

III. ANALYSIS METHODS

A. Measurement procedure

All tone interonset intervals (IOIs) were hand measured by the author using a waveform editing program. Each performance was low-pass filtered at 4.9 kHz and digitized at a 10-kHz sampling rate. The digitized waveform was displayed on the screen of a computer terminal, 2 s at a time. The screen resolution for that display was about 2 ms. A cursor was placed at each tone onset, and a permanent "label" was attached to that point in the waveform file. The differences between the time points of successive labels yielded the IOIs, which were noted down to the nearest millisecond. After some practice, it took about 2 h to measure one complete performance.

Two problems had to be coped with. One was that the onset of a soft tone was often difficult to detect by eye, particularly in some of the older recordings, which had much surface noise. (DAV was the worst by far.) An auditory method was used in that case: The cursor was moved back in small steps, and the waveform segment up to the cursor was played back until the onset of the tone in question could no longer be heard. This procedure was time-consuming but usually resulted in rather accurate location of tone onsets (cf. Gabrielsson, 1987). The second problem concerned onset asynchronies among simultaneous tones in different voices. Unintended asynchronies, which usually were too small to be detected by eye in the waveform, had to be ignored. Larger asynchronies, which in most cases must have been intended by the artists, were noted (especially in performances by CO1–3, DAV, MOI, and SHE) and measured, but the label from which the IOI was computed was placed at the onset of the major melody tone (usually the one with the highest pitch and the latest onset). The same convention

TABLE I. The artists and their recordings. Abbreviations: CD = compact disc; C = cassette; < = date of liner notes (recording date probably the same or preceding year); ~ = estimated date; T = "Träumerei" only.

Code	Artist	Recording
ARG	Martha Argerich (1941–)	DG 410 653-2 (CD) [<1983]
ARR	Claudio Arrau (1903–1991)	Philips 420-871-2 (CD) [1974]
ASH	Vladimir Ashkenazy (1937–)	London 421 290-2 (CD) [1987]
BRE	Alfred Brendel (1931–)	Philips 9500 964 [<1980]
BUN	Stanislav Bunin[a]	DG 427 315-2 (CD) [1988]
CAP	Sylvia Capova[a]	Stradivari SMC-6020 (C) [T] [<1987]
CO1	Alfred Cortot (1877–1962)	EMI 3C 153-53793M [1935]
CO2	Alfred Cortot	EMI 3C 153-53794M [1947]
CO3	Alfred Cortot	EMI 3C 153-53795M [1953]
CUR	Clifford Curzon (1907–1982)	London LL-1009 [~1955]
DAV	Fanny Davies (1861–1934)	Pearl GEMM CD 9291 (CD) [1929][b]
DEM	Jörg Demus (1928–)	MHS OR 400 [~1960s]
ESC	Christoph Eschenbach (1940–)	DG 2535 224 [<1966]
GIA	Reine Gianoli (1915–1979)	Ades 13.243-2 (CD) [1974]
HO1	Vladimir Horowitz (1904–1989)	RCA LD 7021 [T] [1947]
HO2	Vladimir Horowitz	Columbia MS 6411 [<1963]
HO3	Vladimir Horowitz	Columbia MS 6765 [1965] (live) [T]
KAT	Cyprien Katsaris (1951–)	Telefunken 6.42479 AP [1980] (live) [T]
KLI	Walter Klien (1928–1991)	Allegretto ACS 8023 (C) [T][c]
KRU	André Krust[a]	MHS 1009 (orig. Erato) [~1960s]
KUB	Antonin Kubalek (1935–)	Dorian DOR-90116 (CD) [1988]
MOI	Benno Moiseiwitsch (1890–1963)	Decca DL 710,048 [~1950s]
NEY	Elly Ney (1882–1968)	Electrola WDLP 561 [~1935]
NOV	Guiomar Novaes (1895–1979)	Vox PL 11.160 (orig. PL8540) [<1954]
ORT	Cristina Ortiz (1950–)	MCA Classics MCAC-25234 (C) [<1988]
SCH	Artur Schnabel (1882–1951)	Pathé COLH 85 [1947]
SHE	Howard Shelley[a]	CHAN 8814 (CD) [<1990]
ZAK	Yakov Zak (1913–1976)	Monitor MC 2039 [~1960]

[a] Birthdate not known.
[b] Originally Pearl CLA 1000.
[c] Date unknown.

was followed in the case of the written-out *arpeggi* in bars 2, 6, and 18.

B. Data representation

The data were submitted to further analysis in the form of eighth-note IOIs. That is, intervals involving grace note onsets (in bars 2, 6, 8, 16, and 18) were omitted from the data and were considered separately.[1] IOIs longer than a nominal eighth-note were divided into eighth-note intervals of equal duration. Thus, for example, the half-note IOI in bar 2 was represented as four equal eighth-note IOIs. A complete performance thus yielded 254 eighth-note IOIs.

C. Measurement error

The measurement procedure would have been prohibitively time-consuming if maximum accuracy had been aimed for (e.g., by displaying shorter waveform segments on the computer terminal screen). A certain amount of speed-accuracy tradeoff had to be taken into account. The magnitude of the resulting measurement error can be estimated from three performances (BRE, CO2, and the first half of CUR) that, for various reasons, had to be remeasured. Two of them (BRE, CUR) derived from good LPs with a moderate amount of surface noise, whereas CO2 was of older vintage and had special problems related to the pianist's tendency to play the left- and right-hand parts asynchronously. There were several very large discrepancies (in excess of 80 ms) between the two measurements of the CO2 performance, due to a conscious change in the author's criteria for treating asynchronies; these discrepancies were not considered true measurement errors and were omitted from the data presented below, though some smaller inconsistencies of the same kind may still be included.

The measurement error distributions are shown in Table II. The combined BRE + CUR data represent average accuracy, whereas the CO2 data constitute a worst-case scenario; only DAV was even more difficult to measure. It can be seen that over 90% of the measurement errors in the BRE + CUR data did not exceed 10 ms, which is less than 2% of the average IOI. The largest error was under 35 ms, or about 6% of the averge IOI, and the average measurement error was 4.3 ms, less than 1%. In the CO2 data set, about 90% of the errors ranged from 0 to 20 ms, and the largest error was under 50 ms. The average error was 10.4 ms, or about 2%. Measurements of the virtually noiseless CD recordings presumably were even more accurate than suggested by the BRE + CUR data.

TABLE II. Distributions of absolute measurement error in two sets of remeasured data.

Range (ms)	BRE + CUR ($N=412$) Percent	BRE + CUR ($N=412$) Cum.	CO2 ($N=245$) Percent	CO2 ($N=245$) Cum.
0–5	77.4	77.4	24.5	24.5
6–10	13.8	91.3	40.0	64.5
11–15	2.7	93.9	18.8	83.3
16–20	1.7	95.6	5.7	89.0
21–25	2.2	97.8	3.7	92.7
26–30	1.7	99.5	3.3	95.9
31–35	0.5	100.0	2.0	98.0
36–40			1.2	99.2
41–45			0.4	99.6
46–50			0.4	100.0

IV. RESULTS AND DISCUSSION

The strategy in presenting the results will be to proceed from more global properties to more detailed aspects.

A. Overall tempo

Perhaps the most obvious dimension along which different performances vary is that of overall tempo. The present set of performances is no exception. Tentative estimates of global tempo were obtained by computing the first quartiles (the 25% point) of the individual eighth-note IOI distributions, multiplying these millisecond values by 2, and dividing them into 60 000. The choice of the first quartile was motivated by the consideration that expressive lengthening of IOIs is both more frequent and more pronounced than shortening, and also by the fact that it gave tempi for two pianists, BRE and DAV, that agreed closely, respectively, with Brendel's (1981) statement of his preferred tempo and with Clara Schumann's (DAV's teacher's) recommended tempo. (See Repp, in preparation, for details.) Table III shows that the tempo range extended from 48 to 79 quarter-notes per minute. Apart from the fact that the three fastest performances are all old recordings, there does not seem to

TABLE III. Overall tempi (qpm) estimated from the first quartile of the IOI distribution.

48	ESC
49	
50	KAT, NEY
51	
52	ZAK
53	CAP
54	KLI
55	
56	CUR
57	BUN, NOV
58	DEN, KUB
59	ARR, KRU, MOI, SCH
60	
61	HO2
62	ARG
63	ASH
64	HO3
65	HO1, SHE
66	CO3
67	BRE, ORT
68	GIA
69	
70	
71	
72	CO1
73	
74	
75	CO2
76	
77	
78	
79	DAV

be any systematic relationship between tempo and the time at which the recording was made, nor with pianists' gender, age at the time of recording, or country of origin.

B. Repeats

The first step in the data analysis was an attempt to eliminate redundancy and reduce random measurement error by averaging across repetitions of the same (or highly similar) musical material. Of course, such averaging is meaningful only if there are no systematic differences in timing microstructure across repeats. The prime target for averaging was the first period, bars 1–8, which was repeated literally, according to Schumann's instructions in the score. All but two pianists (DAV, KRU) observed this repeat.[2]

To compare the first and second repeats for all pianists, a *grand average timing pattern* was first obtained by computing the geometric mean of corresponding IOIs across all 28 performances. (For DAV and KRU, the data of bars 1–8 were simply duplicated for the missing second repeat.) The geometric mean was preferred over the arithmetic mean because it compensated for any tendency of slower performances to show more expressive variability, which would have dominated in an arithmetic average. In this grand average, the two repeats of bars 1–8 were found to have extremely similar timing profiles, with only a slight tendency for the second repeat to be played slower. The correlation between these two averages was 0.987, which indicates that any variations across repeats for individual pianists were either random or idiosyncratic. The correlations between the two repeats in individual performances are shown in Table IV (column 2); most of them were quite high. The close similarity of timing patterns across repeats has been noted in virtually every study in which such a comparison was made (see, e.g., Seashore, 1938; Palmer, 1989; Repp, 1990). Thus it seemed justified to average across the two repeats of bars 1–8 in all further analyses, except as noted.

There were other instances of identical or highly similar musical material in the piece. Their timing patterns may be compared in Fig. 3, which plots the grand average eighth-note IOIs. A logarithmic ordinate scale is used here to accommodate the longest values; it also makes the scale comparable to the percentage scale used by authors such as Gabrielsson (1987). The abscissa represents metrical distance (or "score time") in eighth-note steps.[3] Longer notes appear as "plateaus" of multiple eighth-notes; that way, all IOIs are represented on the same proportional scale.

The left-hand panel of Fig. 3 compares the timing profiles for the three type-a phrases. Bars 17–20 (phrase a1) are musically identical with bars 1–4 (see Fig. 1), and it can be seen that their timing patterns are highly similar ($r = 0.986$), though bars 17–20 tend to be played at a somewhat slower tempo. The individual correlations for these four bars (Table IV, column 3) are comparable to the correlations between the two repeats of bars 1–8. The timing pattern for bars 21–24 (phrase a2) is initially similar but diverges soon, due to the *fermata* (long hold) in bar 22 and the progressive *ritardando* (slowing of tempo) toward the end of the piece.

The right-hand panel of Fig. 3 compares the timing patterns of the three type-b phrases. Once again a striking similarity can be seen, due to the similarity or identity of the melodic structure (see Fig. 2). The timing profiles of phrases b2 (bars 9–12) and b3 (bars 13–16) are especially close. Only the (prescribed) *ritardando* in bar 16 is much more pronounced than that in bar 12 (which is not notated). If the final halves of bars 12 and 16 (the *ritardandi*) are excluded, correlations between these two timing patterns are again extremely high (see Table IV, last column). Phrase b1 (bars 5–8) also shows a rather similar profile; differences occur precisely where its musical content deviates from phrases b2 and b3 (especially in bar 6).

TABLE IV. Correlations of the timing profiles for repeated or similar sections (R = repeat; GM = geometric mean).

Bars:	1–8/1R–8R	1–4/17–20	9–12/13–16
ARG	0.781	0.861	0.630
ARR	0.933	0.938	0.850
ASH	0.922	0.951	0.927
BRE	0.953	0.888	0.880
BUN	0.510	0.633	0.632
CAP	0.935	0.922	0.846
CO1	0.859	0.717	0.837
CO2	0.865	0.736	0.851
CO3	0.750	0.741	0.571
CUR	0.937	0.911	0.921
DAV	*	0.863	0.863
DEM	0.938	0.924	0.828
ESC	0.890	0.734	0.802
GIA	0.777	0.809	0.914
HO1	0.918	0.958	0.697
HO2	0.811	0.861	0.822
HO3	0.826	0.738	0.770
KAT	0.825	0.901	0.867
KLI	0.804	0.893	0.861
KRU	*	0.875	0.940
KUB	0.951	0.913	0.888
MOI	0.805	0.855	0.493
NEY	0.920**	0.904	0.872
NOV	0.931	0.906	0.908
ORT	0.677	0.609	0.707
SCH	0.676	0.574	0.748
SHE	0.868	0.831	0.859
ZAK	0.839	0.875	0.909
GM	0.987	0.986	0.950

*no repeat, **bars 1–4 only (see footnote 2).

The correlations in Table IV indicate which artists were highly consistent in their timing patterns, and which of them were less consistent. The consistent group is led by ASH, CUR, and NOV and includes many others; the less consistent group includes ARG, BUN, CO3, MOI, ORT, and SCH.[4]

C. Melodic/rhythmic structures and the average timing pattern

Let us examine now the average timing pattern and its relation to the musical structure in greater detail. Although the grand average timing profile is not necessarily an optimal performance timing pattern, it captures features that many individual performances have in common, whereas it suppresses random and idiosyncratic timing deviations that differ from performance to performance.

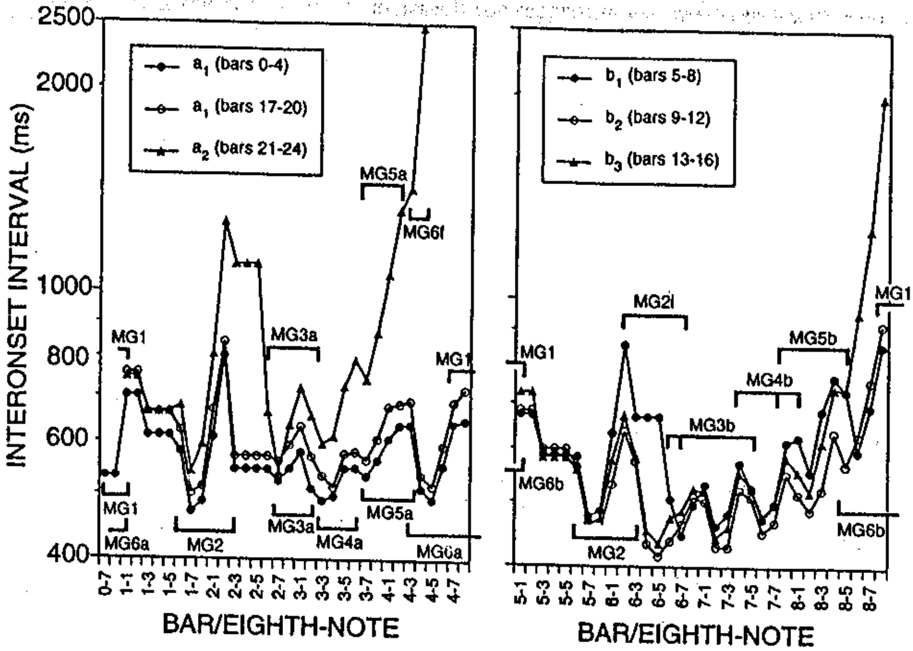

FIG. 3. Grand average IOIs (geometric means across all 28 performances). The two panels show the timing profiles for phrases of type a and b, respectively. Primary melodic gestures (cf. Fig. 2) are indicated by brackets. The bar numbers on the abscissa refer to bars 0–8 only; for the later phrases, an appropriate constant must be added.

In Fig. 3, it can be seen that there are global temporal trends within phrases, particularly those of type b. Their timing profiles follow a concave function on which various local peaks are superimposed. Bars 21–24 (phrase a2), too, seem to follow such a curve, though it is grossly distorted by the *fermata* and the final grand *ritardando*. Bars 1–4 and 17–20 (phrase a1) have a flatter global timing shape. The curvilinear trends in type-b phrases are reminiscent of the parabolic curves hypothesized by Todd (1985) as the basic phrasal timing pattern. Todd's model,[5] however, operates on the durations of time units equivalent to whole bars, whereas the present timing profiles are based on eighth-note units and thus provide much finer resolution. The curves that "cradle" the present timing patterns (i.e., the "bottom lines" of the type-b timing profiles) are, in fact, poorly fit by quadratic functions. However, they do represent the general principle formalized by Todd and observed by many others, that there is a slowing of the tempo at major structural boundaries, in proportion to the importance of the boundaries. Thus the most extreme *ritardando* (prescribed in the score) occurs at the end of the piece; a pronounced slowing down (also prescribed) occurs at the end of the second period (bar 16); next come the end of the first period (bar 8) and the end of phrase b1 (bar 12), which show about the same amount of *ritardando* (not explicit in the score); the end of phrase a1 (bars 4 and 20) shows the smallest, but still quite noticeable slowing down.

The piece begins with a brief melodic gesture (MG1) that ascends from the dominant to the tonic, with accent on the latter (hence the intervening bar line, which indicates accent on the following beat). MG1 recurs in bars 4–5, 8–9, 12–13, 16–17, and 20–21, though the upbeat is shortened in bars 8, 12, and 16 (cf. Fig. 1). It thus contains a single IOI of variable nominal length. Its realization was strongly context dependent: The quarter-note upbeat at the beginning of the piece (bar 0) was short relative to the following IOI, which seemed fairly stable across all phrases. The quarter-note upbeats at the ends of bars 4 and 20, which coincided with MG6a, were relatively longer. The eight-note upbeats in bars 8 and 12, which coincided with MG6b, were even longer. Finally, although the grace-note upbeat of bar 16 is not shown explicitly in Fig. 3, the IOI accommodating it was lengthened enormously, due to the major *ritardando* at the end of period B. (More information about the grace-note upbeat is provided later on.)

The chord following MG1 enriches the harmonic and rhythmic texture but is not really part of the melody; however, it may be thought of as a kind of echo of the tonic and thus could be linked with MG1. The chord bisects the intergesture interval between MG1 and MG2. Of the resulting two IOIs, the first (nominally shorter) one was consistently longer relative to the second (nominally longer) one, as if pianists tried to equalize the two. This may reflect final (and/or accent-related) lengthening of the tonic in MG1, which is absorbed by the IOI preceding the chord.

MG2 is undoubtedly the most salient melodic gesture of the piece. It comprises five eighth-notes that ascend in increasingly larger pitch steps to a final note which constitutes the apex of the four-bar melodic arch that constitutes each phrase. Variants of the gesture occur in bars 1–2, 5–6, 9–10, 13–14, 17–18, and 21–22. MG2 is unusual because it culminates on the second beat of a bar, negating entirely the customary accent on the first beat. As can be seen in Fig. 3, the average temporal pattern of the five IOIs is quite similar in all occurrences of the gesture: The first IOI is close to the proportional duration of the preceding IOI, but the next two

IOIs are much shorter. The fourth IOI is longer again, similar to the first, and the fifth IOI is the longest; it is clearly visible as a sharp peak in the timing profile. The duration of that fifth IOI varies with position in the piece: It is least extended in bars 10 and 14, longer in bars 2, 6, and 18, where it accommodates the two grace notes of the left-hand chord (cf. Fig. 1), and longest in bar 22 where it leads to the climactic *fermata*. In fact, this eighth-note IOI in bar 22 tended to be lengthened relative to the *fermata* chord itself (the plateau following the peak). The specific *accelerando-ritardando* shape of MG2 will be examined more closely later on.

The IOI following MG2 in bars 2, 6, 18, and 22 (plateau in Fig. 3) represents another intergesture interval, nominally three or four eighth-notes long. Relative to the last IOI of MG2, this interval was much shorter proportionally, even in bar 22, where it received the *fermata*. The *fermata* had its maximal effect on the intergesture interval, doubling its duration relative to bars 2 and 18, but it also affected the whole preceding MG2. In bars 10 and 14, the intergesture IOI is bridged by the inner-voice imitation gesture MG2i, which has a timing pattern that is almost the mirror image of MG2 in that it accelerates during the first three IOIs and slows down only at the end. The first IOI is long because it also represents the last IOI of MG2; the relative length of the second IOI may be transitional, or it may reflect final lengthening of MG2 which was absorbed by MG2i.[6]

The remaining MGs, which descend in a chain from the melodic apex reached by MG2, are patterned differently in phrase types a and b. The grouping structure of the type-a chain is indicated with slurs in the score (Fig. 1). MG3a consists of four (a1) or five (a2) eighth-notes in the soprano voice, with accent on the penultimate; a shorter accompanying gesture occurs in the tenor voice (a1 only). It can be seen in Fig. 3 that lengthening on the accented note occurred in performance. MG4a is similar in rhythmic structure to MG3a and exhibits a similar timing pattern, except that the final unstressed eighth-note IOI was lengthened as well, especially in phrase a2, where the final *ritardando* tilted the timing profile. MG5a, reinforced by shorter secondary MGs in the bass voice, is similar to MG4a, except that in phrase a1 it leads into a final half-note that coincides with the onset of MG6a. Its timing pattern is a progressive lengthening, similar to that observed in MG4a. In phrase a2, the final *ritardando* is in full control. The final gesture in phrase a1, MG6a, is in the bass voice and leads back to the low-pitched tonic which doubles the tonic of the next MG1. Its first IOI coincides with the final lengthened IOI of MG5a, and the last two IOIs coincide with the quarter-note upbeat of MG1 and are both lengthened as well. The IOIs in between are considerably shorter. The total timing profile of MG6a is not unlike that of MG2, which it resembles rhythmically. The final gesture in phrase a2, MG6f, is a truncated version of the preceding MGs and shows an enormous slowdown in tempo; note, however, the discontinuity in the *ritardando* between MG5a and MG6f, which is a manifestation of final lengthening in MG5a.

The melodic chain of type b is also divided into four MGs, but they are organized differently. The slurs in the score are not entirely consistent here; what distinguishes the gestures is the fact that they occur in different voices, imitating and overlapping each other. Simultaneously, other voices articulate shorter two-note cadential gestures which, as can be seen in Fig. 3, essentially govern the timing pattern. In each MG, *both* IOIs constituting these accompanying motifs were lengthened, the first, unaccented one usually more than the second, accented one. This is explained by the harmonic function of the tone cluster initiating the unaccented IOI, which is that of suspense followed by resolution. In part, however, it may also be due to final lengthening of the preceding, overlapping MG.

In summary, these observations illustrate several general principles of performance timing: (1) Whole phrases tend to show global tempo curves characterized by an initial *accelerando* and a more pronounced final *ritardando* whose degree reflects the degree of finality of the phrase in the hierarchical grouping structure. (2) "Riding" on the global pattern, individual MGs often show a similarly curved local pattern, though accent placement, harmonic factors, and the influence of overlapping MGs in other voices may modulate that pattern in various ways. Gesture-final lengthening is commonly observed.

D. Principal components analysis

An alternative way of capturing commonalities among different performances is offered by the statistical technique of principal components (factor) analysis. This method decomposes the data matrix (N performances by M IOIs) into a number of independent components or factors, each of which resembles a timing profile (i.e., M standardized "factor scores"). The original data are approximated by a weighted sum of these factors; the weights, which differ for each performance, are called "factor loadings" and represent the correlations between the performance timing profiles and the factor score profiles. The degree of approximation (the percentage of the "variance accounted for" or VAF) depends on the number of factors that are considered significant; a common criterion employed here is that they should have "eigenvalues" greater than 1. (Eigenvalue $= N$ times an individual factor's proportion of VAF.) The first factor always accounts for the largest amount of variance and represents a kind of central tendency or most common pattern. Additional factors account for increasingly less variance and thus represent patterns shared by fewer performances. A standard technique for simplifying the factor loadings and thereby increasing the interpretability of the factors is called "Varimax rotation." It increases large factor loadings and reduces small ones without changing the number of factors or the total VAF; however, the VAF is redistributed among the rotated factors.

Principal components analysis thus can reveal whether there is more than one shared timing pattern represented in the sample of 28 performances. If there is essentially only one way of performing the piece, then a single factor should explain most of the variance in the data. If there are several radically different timing patterns, then several orthogonal factors should emerge. Idiosyncratic timing patterns will not lead to a significant factor and will constitute part of the variance not accounted for. Since all timing profiles are standardized and intercorrelated in the course of the analysis, differences in overall tempo are automatically disregarded.

Although it would seem desirable to conduct the analysis on the complete performances, the large *ritardandi* observed universally at the ends of phrases a2, b1, b2, and b3, as well as the large slowdown at the *fermata* in phrase a2, dominate the overall timing pattern and cause high correlations among the performances, with the result that the interesting variation among the shorter IOIs is lost. In fact, a principal components analysis conducted on the complete data yielded only a single factor, indicating that all pianists observed the major *ritardandi*, which is not very surprising. Therefore, it was decided to analyze only the data for bars 0–8 (phrases a1 and b1), which did not include any extreme lengthenings (cf. Fig. 3). Because of the parallelism of the timing profiles of these phrases to those of similar type (see Fig. 3), such a restricted analysis essentially captures the whole performance minus the major *ritardandi*.

This analysis yielded four significant factors. Together they accounted for 76% of the variance. For individual performances, the VAF ranged from 60% to 90%. The timing profiles representing the first three factors are shown in Fig. 4, and the factor loadings of the 28 performances are listed in Table V.

a. Factor I (VAF=30%). This factor represents a timing pattern shared (partially) by a large number of performances; half the pianists had their largest loading on this factor, with SCH, ASH, and BRE leading the group. This pattern has the following features: a relatively long initial upbeat (MG1) and pronounced lengthening of the last IOI of MG6, which accompanies the upbeat to the next MG1 (positions 4–8 and 8–8); general lengthening of long IOIs (plateaus in the profile); a relatively long *accelerando* phase in MG2 followed by a pronounced lengthening of its last IOI; and small but regular gestural accent peaks for MG3–MG6, except for special emphasis at the end of MG5b (positions 8–3 to 8–5). This pattern reflects the melodic/rhythmic structure even more clearly than the average timing profile displayed in Fig. 3.

TABLE V. Sorted rotated factor loadings of the 28 performances, bars 0–8. (Loadings smaller than 0.4 are omitted.)

	I	II	III	IV
SCH	0.844			
ASH	0.760	0.423		
BRE	0.738			
DAV	0.695			
CAP	0.695	0.438	0.411	
SHE	0.692			
KAT	0.685			
KUB	0.672			
ARR	0.664	0.463		
ZAK	0.644	0.460		
ORT	0.637			
CUR	0.615		0.429	
KRU	0.593		0.495	0.445
DEM	0.551	0.495		
HO1		0.888		
HO3		0.886		
HO2		0.837		
ARG		0.770		
NEY		0.665		0.524
ESC	0.482	0.626		
GIA		0.595	0.491	
BUN		0.588		
KLI	0.523	0.570		
NOV	0.509	0.549		0.464
CO3			0.876	
CO1			0.875	
CO2			0.850	
MOI	0.501			0.582

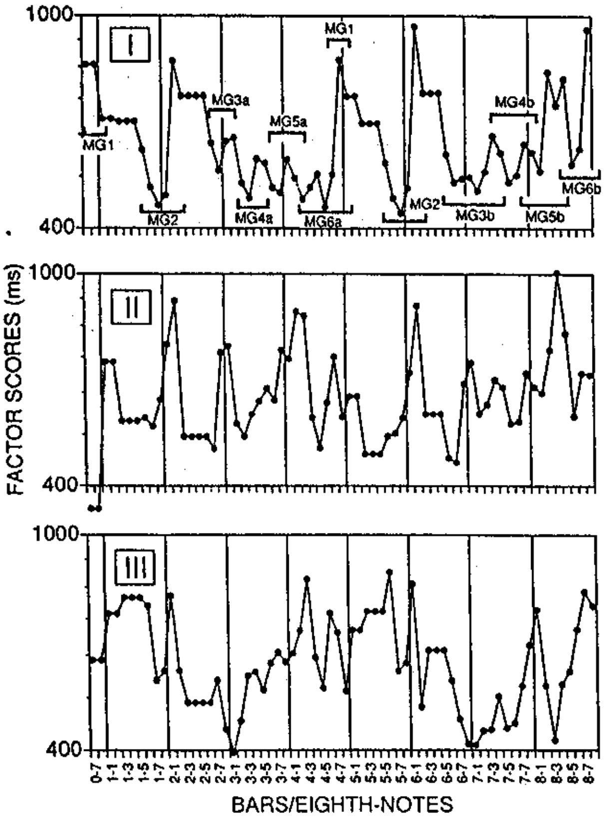

FIG. 4. Timing patterns of the first three principal components for bars 0–8. The standardized factor scores were converted into milliseconds by multiplying them with the average within-performance IOI standard deviation and adding this product to the grand mean IOI. Bar lines and MG brackets are added for orientation.

b. Factor II (VAF=25%). This may be called the "Horowitz factor," since the three performances by that artist showed the highest loadings, though ARG, NEY, ESC, and a number of other performances shared features of this pattern. Due to the statistical orthogonality of the factors, this timing pattern is radically different from that of factor I. It is characterized by a very short initial upbeat (essentially reduced to an eighth-note, though it was rarely that extreme in the actual performances; see below); relatively short long IOIs (i.e., low plateaus); no *accelerando* but a pronounced *ritardando* during MG2; pronounced lengthening in MG3a, MG3b, and MG5b, with extreme prolongation of the IOI in position 8–4, which accommodates a grace note.

c. Factor III (VAF=15%). This is the "Cortot factor;" the three performances by this artist were the only ones that showed substantial loadings. Its timing pattern is most unusual: a relatively short upbeat in MG1; a short final IOI in

MG2 (positions 2–2 and 6–2); marked speeding up during MG3 with a following *ritardando* extending through MG4 to the end of MG5; and rushing during the end of MG5b, followed by a pronounced *ritardando* during MG6b.

d. Factor IV (VAF = 6%). This factor seems less important and will not be discussed in detail, as no artist showed a high loading on it (Table V).

The three factor timing patterns shown in Fig. 4 do not correspond exactly to any real performance, though they are similar to certain performances, as indicated by the factor loadings (Table V). Most artists' performances are best described by a weighted combination of several factors; thus, for example, the CAP timing profile is a combination of factors I, II, and III. Such a combination will of course result in an attenuation of extreme features. About one fourth of the variance was not explained by the four factors extracted, and most of that variance was probably nonrandom, representing the artists' individuality.

It had to be asked at this point whether the Horowitz and Cortot factors emerged only because each of these artists was represented by three different performances.[7] The principal components analysis was repeated with only a single performance of each of these pianists included (CO1 and HO2). The three factors that emerged (VAF = 71%) were highly similar to the first three factors of the earlier analysis. The second and third factors still had their highest loadings in HO2 and CO1, respectively. The patterns of factor loadings for the three factors in the two analyses correlated 0.96, 0.98, and 0.95, respectively. Thus the Horowitz and Cortot factors are not artifacts due to the over-representation of these artists in the sample. They represent two true alternatives to the "standard" pattern of performance timing instantiated by factor I—alternatives that only Horowitz and Cortot dared to choose in nearly pure form, but that several other pianists incorporated partially into their timing strategies.

E. Detailed analyses

In this section, we examine how melodic gestures and other details of the score were executed by using, as it were, a magnifying glass on the data. While the emphasis has so far been primarily on commonalities, the analyses are now directed increasingly toward uncovering artistic diversity. They reveal the different ways in which a gesture may be shaped by pianists of great authority. They also reveal some unusual, and occasionally questionable, interpretations of notational details in the score. In addition to demonstrating the range of individual differences, these analyses illustrate what temporal patterns are preferred by a majority of the artists, and what patterns are never used, and hence presumably unacceptable.

1. MG1

This gesture consists of merely two tones forming the pitch interval of an ascending fourth. It is followed, however, by a chord whose timing, though not strictly part of the melody, is also of interest. MG1 appears six times: Three times with a nominal quarter-note upbeat (bars 0, 4, and 20) and three times with a nominal eighth-note upbeat (bars 8, 12, 16), one of which (bar 16) is written as a grace note and thus is open to the assignment of an even shorter value. Two of the instances of the gesture are relatively unconstrained because the upbeat is unaccompanied (bars 0 and 16); in the other cases the upbeat coincides with eighth-notes in the bass voice (MG6).

Figure 5 presents the individual data in terms of two IOI ratios, $A/(B+C)$ and B/C, where A is the duration of the intragesture IOI (i.e., the upbeat), B is the intergesture IOI preceding the chord (nominally two eighth-notes), and C is the intergesture interval following the chord (nominally three eighth-notes). If the score were played literally, $A/(B+C)$ should be either 0.2 or 0.4, depending on whether the upbeat is notated as an eighth-note or a quarter-note, and B/C should be 0.67. In Fig. 5, however, the ratios have been normalized with respect to the underlying eighth-note pulse (assumed to be isochronous for this purpose), so that the expected "literal" ratios (effectively, tempo ratios) are equal to 1 in all cases.

Let us consider first the ratio plotted on the abscissa. The top row of panels in Fig. 5 shows the three instances in which the MG1 upbeat is nominally a quarter-note. The leftmost panel shows the unconstrained situation at the beginning of the piece (bar 0). It can be seen that very few pianists played this initial upbeat—or the following intergesture inverval, as the case may be—"as written" (ratio of 1). The large majority played the upbeat quarter-note *short* relative to the intergesture interval. Several pianists (NEY, CO1, BUN) come very close to playing it as if it were an eighth-note (ratio of 0.5), and one (ARG) does so without question. This may have been a deliberate anticipation of the (notational) shortening of the upbeat in later incarnations of MG1, but it is a true deviation from the written score and is perceived as such.

Later, however, when the quarter-note upbeat occurs together with two eighth-notes in the bass voice (top center and right-hand panels in Fig. 5), pianists are more evenly divided between those who shorten the upbeat and those who lengthen it relative to the intergesture interval. While the average ratio is close to 1, there is enormous variation among individual artists, and some who severely shortened the initial, unconstrained upbeat (e.g., ARG, BUN) do not repeat this tendency later on.

The lower row of panels in Fig. 5 shows instances where the upbeat is nominally an eighth-note. In two cases (left-hand and center panels) the upbeat is accompanied by a bass-voice eighth-note that marks the approaching end of MG6 and of a whole phrase, so here the overwhelming tendency is to lengthen the upbeat, sometimes to an extent corresponding to a nominal quarter-note (ratio of 2; BUN and ARR in bar 12). In bar 16, where the upbeat is notated as a grace eighth-note, a significant minority of pianists plays the upbeat shorter than an eighth-note, in one case (KAT) shorter than a sixteenth-note (ratio of 0.5). The majority, however, still lengthen it, and at least one (ASH) plays the grace note as if it were a quarter-note (ratio of 2).

There is little indication of clustering or bimodality in these data. Although there are some "outliers," the values of each ratio seem to be rather evenly distributed over a wide

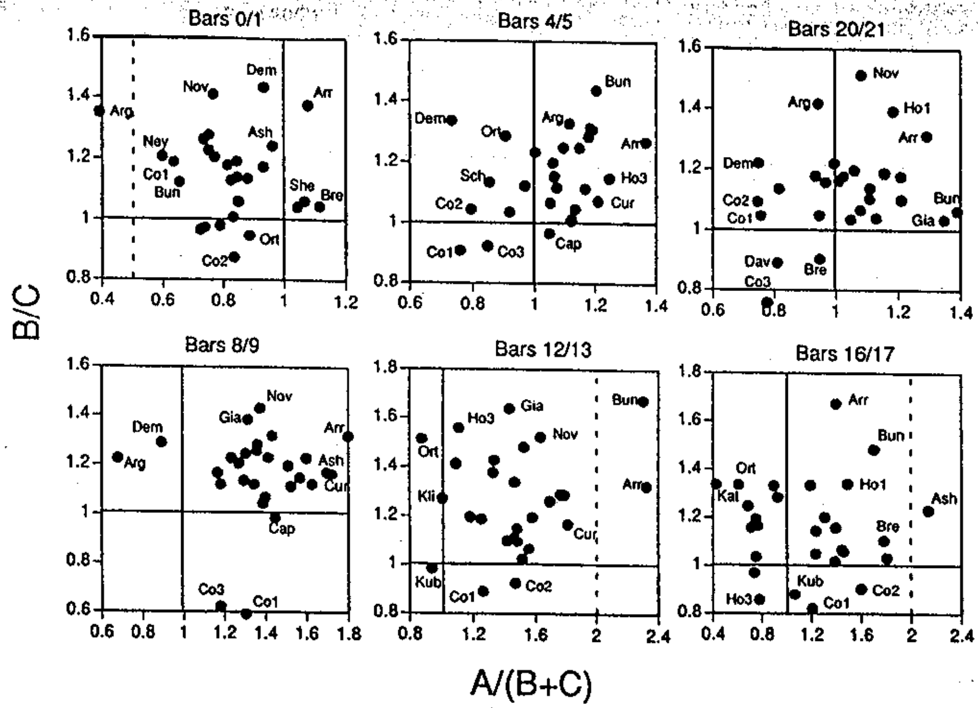

FIG. 5. Timing patterns of the six instances of MG1 and the following chord. (See text for explanation of ratios.) Ratios of 1 represent equal underlying beats (i.e., equal tempo). The upper panels plot the three instances where the upbeat is nominally a quarter-note, while the lower panels plot the three instances where the upbeat is nominally an eighth-note (a grace note in bar 16). The dashed lines indicate a doubling or halving of the nominal duration in execution.

range. Correlations of individual timing patterns across the different instances of MG1 are not high, though some consistencies can be seen. Thus ARR and BUN tend to lengthen the upbeat, whereas ARG, DEM, and CO1–3 tend to shorten it. Some pianists, perhaps to be regarded as "literalists," consistently avoid extremes (e.g., ESC, KRU, MOI, ZAK).

With regard to the B/C ratio, plotted on the ordinate, there is more consistency. In all six instances, which are notationally equivalent, there is a strong tendency to lengthen the rest preceding the chord, sometimes to the extent that the onset of the chord occurs in the middle of the intergesture interval (ratio of 1.5). Among those who tend to play the chord very late are ARG, ARR, BUN, and NOV. Nevertheless, there are some pianists who do not show that pattern, most notably Cortot.

Finally, it should be noted that the two ratios are uncorrelated across different artists. Evidently, the timing of MG1 is quite independent of the timing of the chord in the intergesture inverval.[8]

2. MG2

This is the signature melodic gesture of the piece, and its manner of execution is crucial to the impression of a performance of "Träumerei." It offers a unique opportunity to investigate the temporal shaping of a gesture, for several reasons: (1) It comprises six notes (i.e., five IOIs)—a sufficient number of degrees of freedom for any constraints on temporal shape to emerge very clearly; (2) it is unidirectional in pitch and uninterrupted by metric accents—the accent that would normally occur on the note following the bar line is clearly suspended in this case; (3) it recurs six times, with slight variations; and (4) pianists are likely to give special attention to its execution.

MG2 occurs in bars 1–2, 5–6, 9–10, 13–14, 17–18, and 21–22. (We will not consider the imitation gesture MG2i in detail.) The versions in bars 1–2 and 17–18 are identical; during the last IOI, two grace notes (a written-out *arpeggio*) occur in the left-hand accompaniment of the melody. In bars 5–6 and 21–22, the penultimate pitch interval is extended from a fourth to a major sixth; in addition, the melody in bars 21–22 leads to a *fermata* and also lacks the grace notes during the last IOI. In bars 9–10 and 13–14, there are no grace notes, the penultimate melodic interval is reduced to a minor third in the first instance, there is a change of key (to B-flat major) in the second instance, and the occurrence of MG2i in the middle voices creates a forward movement that is absent in the other variants. Therefore, timing differences are to be expected reflecting these factors.

The average temporal shapes of the six versions of MG2 are shown in Fig. 6. The five IOIs for each version, representing the geometric means across all 28 performances, are plotted here on a linear scale. The five data points for each version have been fitted with a quadratic function (i.e., a parabola), which seems to describe their temporal shape rather well.[9] Each gesture accelerates initially and then slows down at the end. This final *ritardando* is least pronounced in bars 9–10 and 13–14; it is more evident in bars 1–2, 5–6, and 17–18; and in bars 21–22, preceding the *fermata*, it is dramatic. Except in this last instance, there is a tendency for the penultimate IOI to rise slightly above the parabolic trajectory. This was evidently due to including in the average

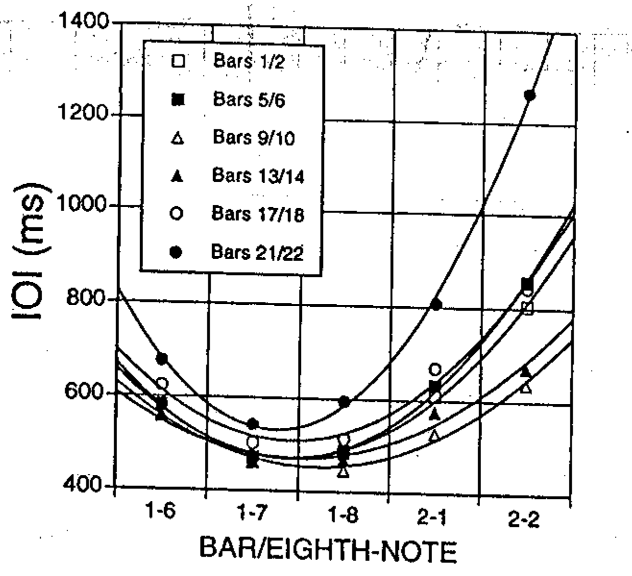

FIG. 6. Geometric mean IOIs for the six versions of MG2, with best-fitting quadratic functions. The abscissa labels refer to bars 1 and 2.

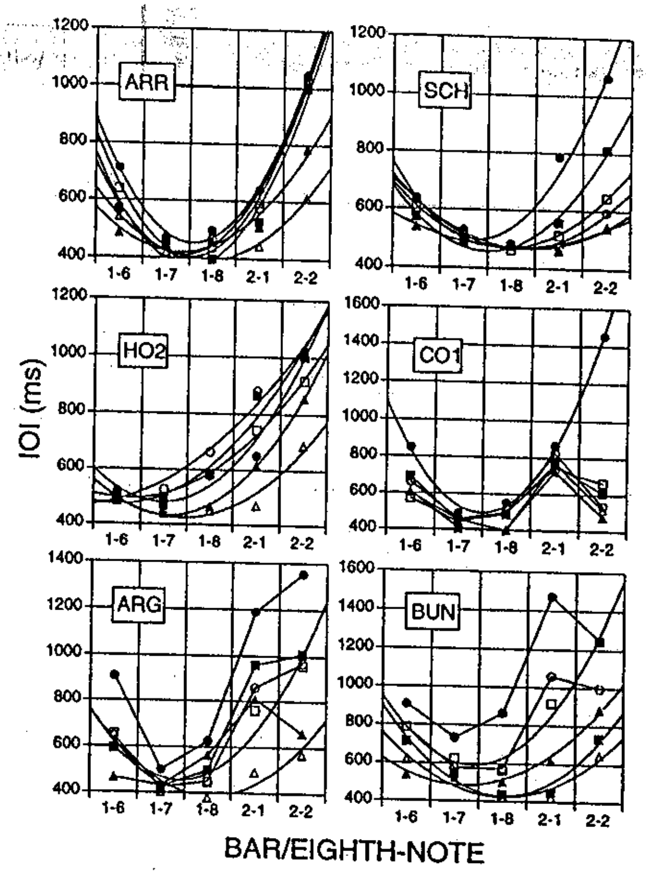

FIG. 7. MG2 timing data for six individual artists, with best-fitting quadratic functions. Where the pattern deviated markedly from a parabolic shape, the data points are connected with straight lines. See Fig. 6 for the legend of symbols.

performances such as Cortot's which, as we have seen in Fig. 4 (factor III), showed a pronounced tendency to shorten the last IOI.

The temporal shapes of the six versions of MG2 were examined and fitted with quadratic functions in each of the 28 individual performances (a total of 168 instances). It emerged that most individual artists' timing patterns (87% of all instances) could be described well by parabolas, with fits ranging from good to excellent. What varied between artists and between different instances of MG2 were the curvature and elevation of the parabolic functions, but not so much their goodness of fit (however, see footnote 9). The data of six individual artists are shown in Fig. 7.

The two panels on top illustrate two individual cases that were fit well by parabolas but differed in curvature: The highly modulated timing curves of ARR contrast with the flatter ones of SCH. The center panels show one representative performance of each of the two great individualists, Horowitz and Cortot. Horowitz's curves are fit fairly well by parabolas but generally lack the initial acceleration shown by most other pianists. Cortot shows a truly deviant pattern: In all three performances (which spanned 18 years!), he shortened rather than lengthened the last IOI. This seems to indicate that he grouped the penultimate tone with the following long tone, treating it like an upbeat; in fact, the first four IOIs are fit well by a parabola. This idiosyncratic timing shape held only for the first five instances of MG2, however; in bars 21–22, preceding the *fermata*, Cortot produced a beautiful parabolic timing shape in all three performances. Finally, as can be seen in the bottom panels of Fig. 7, ARG and BUN, two highly variable pianists, intermittently adopted the alternative timing pattern instantiated by Cortot; one other pianist who did so was CUR. The only other type of deviant timing pattern was shown by ORT who, in bars 9–10 and 13–14 only, lengthened the third IOI above the parabolic trajectory, which led to a W-like shape.

The quadratic functions ($y = C + Lx + Qx^2$) that fit the large majority of individual phrasal shapes are characterized by three parameters: a positive constant C that reflects vertical displacement, related to overall tempo; a generally negative coefficient L that reflects horizontal–vertical displacement of the curve in x–y coordinate space;[10] and a positive coefficient Q that reflects the degree of curvature of the concave (U-shaped) parabola. A question of great theoretical interest was whether there are any constraints among the three parameters; in principle, of course, they are quite independent of each other. The coefficients L and Q were plotted against each other for all 146 individual gestures that followed a parabolic shape. There was a remarkably tight linear relationship between these two parameters, with correlations ranging from 0.93 to 0.98 across the six versions of MG2. The reason for this result is that the location of the minimum of the function is relatively fixed. Therefore, as the curvature of the parabola increases, the negative slope of the tangent at $x = 0$ also increases.

If the x coordinate of the minimum is relatively fixed, then the constant C must be correlated with the y intercept of the function and should increase as a function of curvature Q. There were indeed positive linear correlations between C and Q, ranging from 0.79 to 0.93 (see footnote 11). Thus both L and C are fairly predictable from Q, which leads to the conclusion that the expressive timing patterns of the large majority of the performances of MG2 can be characterized by a single family of parabolas, varying in Q only. This family of functions is shown in Fig. 8; it was generated by increasing Q from 20 to 140 in steps of 20, and by setting L and C according to their average linear regressions on Q. Figure 8 embodies a constraint on the expressive timing shape of MG2 that was obeyed by the majority of pianists. With few exceptions, any individual execution of this phrase can be characterized by a single curvature parameter, with some additional freedom in global tempo (vertical displacement), which is not represented in the figure.

To give some impression of individual differences in tempo modulation during MG2, Fig. 9 plots the average Q coefficient for bars 1–2, 5–6, and 17–18 (which were generally executed similarly; cf. Fig. 6) against that for bars 21–22, with individual artists identified. (Cortot is excluded from this plot.) Most pianists employed moderate modulation in the earlier instances of MG2, but gave a very expressive shape to the last instance in bars 21–22, which led to the *fermata*. Some, most notably DEM, used strong modulation in all instances. Horowitz's three performances are at the other extreme. They appear relatively unmodulated despite a substantial *ritardando* because he did not show an initial *accelerando*; therefore, his timing patterns were fit by parabolas with low curvature.

This "parabolic constraint" on the timing shape of an expressive gesture is of great theoretical interest, as it supports Todd's (1985) suggestion of the parabola as a basic timing function. It also appears to be consistent with his more recent modeling (Todd, 1992b), even though he proposed that a linear function can account for most of the timing variation; clearly, a quadratic function did even better. The parabolic constraint may thus be understood as an allusion to physical motion (cf. Todd, 1992b), which most performers aim at, and which listeners with some musical education find aesthetically pleasing (Repp, 1992b).

Since MG1, the following intergesture interval, and MG2 may form a coherent half-phrase that is planned and

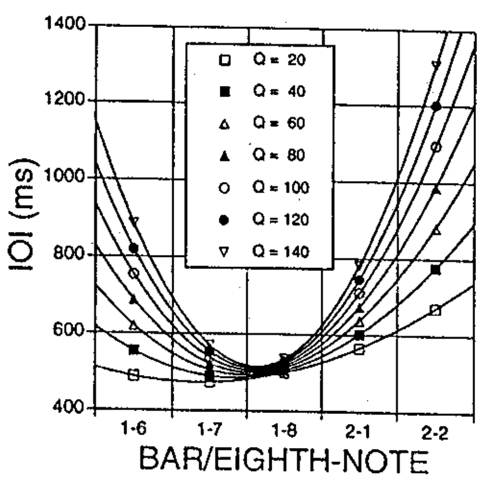

FIG. 8. Family of quadratic functions that describes the timing constraint on MG2 observed by the large majority of pianists. The functions vary in curvature (Q), roughly over the observed range. The other coefficients of the quadratic functions were obtained by the average regression equations. Vertical displacement should be considered an additional free parameter.

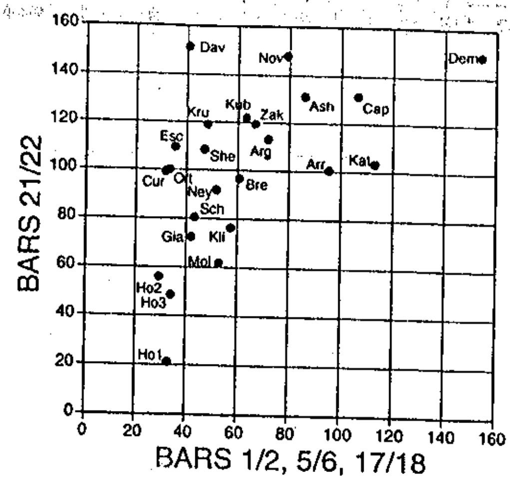

FIG. 9. Individual variations in tempo modulation during MG2. The average curvature (Q) for three instances of the gesture is plotted against the curvature for the last instance (bars 21–22). Only cases following the parabolic timing shape are included.

excecuted as a single expressive unit, their timing relationships across the 28 performances were examined (in bars 0–2 only). None of the three coefficients characterizing the parabolic shape of MG2 correlated significantly with either of the two IOI ratios considered in connection with MG1 (Fig. 5). However, the total interval preceding the onset of MG2 corelated moderately (0.59–0.72) with all three MG2 coefficients. Thus the slower the initial tempo, the slower and the more inflected was MG2 (see, also, footnote 8).

3. Grace notes accompanying MG2

Only three of the six instances of MG2 have grace notes (really a written-out *arpeggio*) in the left hand during the last IOI; they occur in bars 2, 6, and 18. Since all six instances of MG2 followed a parabolic trajectory, on the average (cf. Fig. 6), the grace notes as such were not responsible for the lengthening of the final IOI and the systematic constraints that it followed. Rather, they seemed to be fitted into whatever temporal shape the pianists chose for the primary melodic gesture. We now turn our attention to the execution of these grace notes, which exhibited surprising variability.

In the score, the grace notes are represented as nominal sixteenth-notes. However, since the preceding notes in the left hand are nominally (tied-over) quarter-notes, there is literally no time for the grace notes in the rhythmic scheme; thus they must be "taken away" from the preceding or following note values. The standard way of execution suggested by the spatial placement of the grace notes in the score is to play them within the last IOI of MG2—that is, after the onset of the penultimate melody tone in the soprano voice, but before the onset of the final long tone and the accompanying two-tone chord. However, only 12 of the 24 pianists consistently played the passage according to this conventional interpretation of the notation. (The two repeats of bars 1–8 were examined separately in this analysis.) It seems that this brief passage offered pianists ample opportunity to indulge in "deviations from the score."

Most of the variants are illustrated schematically in Fig.

10, which uses horizontal displacement of the critical notes in bar 2 to indicate the actual succession of tones. The standard version is shown in (a). As shown in (b), some pianists delayed the last note in the left hand until after the octave chord in the right hand: ARR (slightly, bar 6 only), DAV (bar 2 only), MOI (bar 6, once in bar 2), NOV (only the first time in bar 2, but very dramatically), and ZAK (consistently). Others (c) played all notes of the left hand before those of the right hand: ARG (except the second time in bar 6), HO1 and HO2 (bar 2). In a variant of that pattern, the right-hand chord was arpeggiated (DEM, consistently) or mysteriously interleaved with the broken chord in the left hand (Horowitz, in bar 6 only, but in all three performances).

In all instances mentioned so far, the onset of the last melody tone of MG2 did follow the two grace notes, so the grace notes were within the last IOI of the gesture. This made it possible to ask about the relative timing of the grace notes within the IOI (see below). In some further cases, however, one or both of the grace notes coincided with the final melody tone. Thus, SCH consistently played the right-hand octave tones simultaneously with the first grace note (d). BUN (bars 2 and 18) and KUB (bars 2 and 6, second repeat only) played the right-hand tones simultaneously with the second grace note (e). Finally, Cortot (bar 6, in all three performances; also bar 18 in CO1) played all tones simultaneously as a single chord (f), as prescribed only in bar 22 of the score. (BUN also did this once in bar 6.[12])

To investigate the relative timing of the grace notes within the last IOI of MG2, provided they did fall within that IOI [cases (a)–(c) in Fig. 10], the IOI was divided into three parts (A, B, and C), defined by the respective tone onsets. Two interval ratios were calculated: $A/(B+C)$, which indicates the relative delay of the onset of the first grace note, and B/C, which represents the duration of the first grace-note IOI relative to the second. If the first melody tone and the two grace notes were played as a triplet, for example, the two ratios would be 0.5 and 1, respectively; if the grace notes were played as thirty-second notes following a sixteenth-note rest, the ratios would both be 1.

Figure 11 plots $A/(B+C)$ against B/C for all instances where these ratios could be determined. (Bars 2 and 6 are each represented by a single ratio, averaged across the two repeats.) A wide range of ratios was found, but most data points form a cluster in the lower left quadrant, suggesting that the majority of pianists agreed on a particular timing pattern. The central tendency of that cluster is around $A/(B+C) = 0.4$ and $B/C = 0.5$. This means that the grace notes were not played evenly: The first grace note started about 40% into the IOI, and the second grace note was about

(a)
(b)
(c)
(d)
(e)
(f)

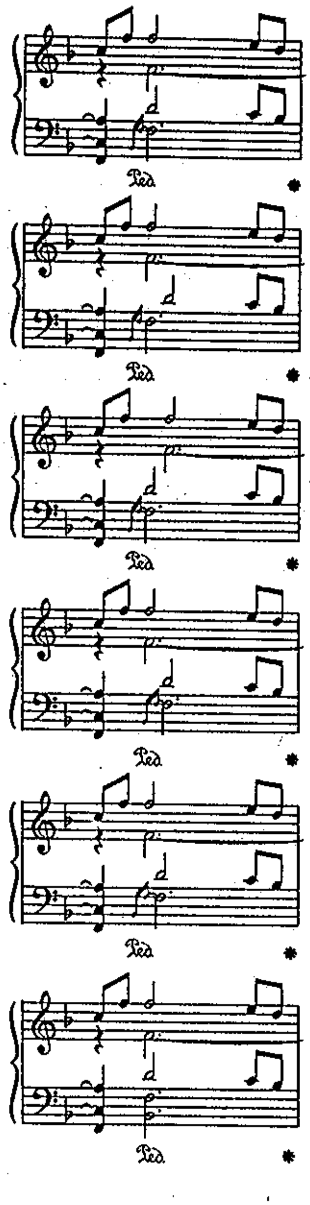

FIG. 10. Schematic illustration of six observed manners of execution of the grace notes accompanying MG2. The notation for bars 2 and 18 is shown, but it stands for bar 6 as well. Spatial displacements of the notes symbolize temporal displacements.

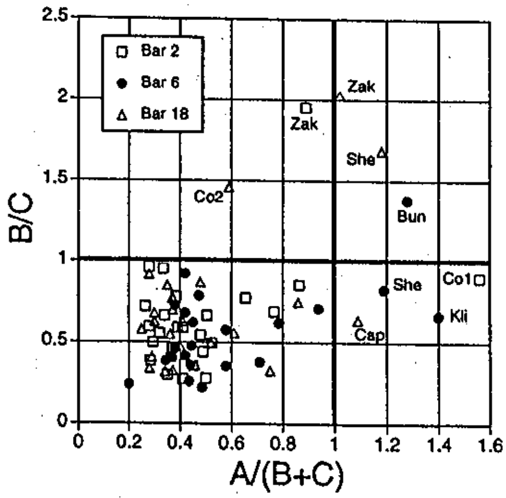

FIG. 11. Relative timing of the grace notes within the last IOI of MG2, for cases (a)–(c) in Fig. 10. The abscissa shows the ratio of the time before the onset of the first grace note (A) and the remainder of the IOI ($B+C$); the ordinate shows the ratio of the time between the two grace note onsets (B) and the time from the onset of the second grace note to the end of the IOI (C). Ratios for bars 2 and 6 are averaged over the two repeats.

twice as long as the first, which means it started about 60% into the IOI, on the average. Considering the compressed scale of the ordinate in the figure, however, it is clear that there was wide variation in the relative durations of the two grace notes, even within the cluster of relative conformity. (Some of this variation undoubtedly reflects measurement error magnified by ratio calculation.) What was almost always true, however, is that the first grace note started during the first half of the IOI [$A/(B+C)<1$], and the second grace note IOI was longer than the first ($B/C<1$). There were some notable exceptions to this pattern, though no individual pianist was consistently deviant. Assuming that these instances are not simply slips of motor control, it would be interesting to know what led individual artists to vary their timing patterns in these ways.

4. MG3a-MG6a

We turn now to a closer examination of the MG chains making up the second half of each phrase. The type-a chain occurs in identical form in bars 2–4 and in bars 18–20 (see Fig. 2). It also occurs in varied and abbreviated form in bars 22–24. We will not deal specifically with MG3a and MG3b in bars 22–23, which on the whole were played as in bars 2–3 and 18–19. MG5a and MG6f in bars 23–24, which carried the final large *ritardando*, will be considered separately later on.

As noted earlier, the average timing profiles for bars 2–4 and 18–20 were extremely similar, though bars 18–20 were played at a somewhat slower tempo overall (see Fig. 3). In the principal components analysis conducted on bars 0–8, the first factor (see Fig. 4, top panel) exhibited a regular sequence of peaks for MG3a–MG6a, which reflect lengthening of accented tones and MG-final IOIs. Detailed examination of the individual timing patterns, however, revealed enormous variety, though not without constraints. No two pianists' interpretations of this MG chain were quite the same, and some artists also changed their patterns significantly between bars 2–4 and bars 18–20. (The two repeats of bars 2–4 will not be considered separately here, though some pianists played even these differently.)

To explore this variability in a reasonably economical way, principal components analysis was used once again, but this time only on the $2\times 18=36$ IOIs comprising MG3a–MG6a in bars 2–4 and 18–20, concatenated. From this analysis, six significant factors emerged—more than from the earlier analysis of bars 0–8, where the larger timing deviations in MG1 and MG2 dominated the correlation structure. Thus there were at least six distinct (i.e., uncorrelated) timing patterns underlying the variation in the data; these patterns are shown in Fig. 12. None of them is fully representative of any individual performance, however, and together they account for only 81% of the variance. The factor scores for bars 2–4 and 18–20 were highly similar and are superimposed in the figure. The factor loadings of the individual performances are shown in Table VI.

a. Factor I (VAF=18%). This factor is characterized by acceleration during MG3a; a fast traversal of MG4a; a dramatic *ritardando* within MG5a with maximal lengthening of the final, unaccented note (position 4–2), further augment-

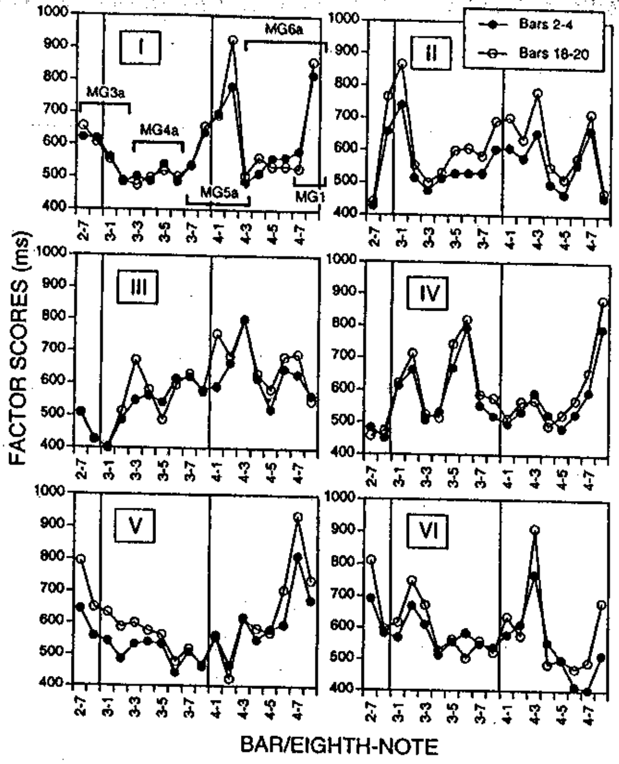

FIG. 12. Rescaled factor scores (underlying timing patterns) for the MG chain of type a in bars 2–4 and 18–20. The abscissa labels refer to bars 2–4.

TABLE VI. Sorted rotated factor loadings from the principal components analysis of MG3a–MG6a. (Loadings below 0.4 are omitted.)

	I	II	III	IV	V	VI
ARR	0.773					
NEY	0.758					
BRE	0.744					
ASH	0.708					
KAT	0.671					0.437
ESC	0.606	0.416				
HO3		0.908				
HO2		0.893				
HO1		0.830				
ARG		0.629	0.402			
CO1			0.873			
CO2			0.832			
CO3			0.784			
ORT		0.446	−0.593			
KUB				0.823		
NOV				0.808		
SCH				0.786		
CUR				0.676	0.541	
MOI					0.864	
SHE	0.564				0.693	
DAV				0.456	0.574	
GIA		0.410	0.535		0.564	
KLI		0.576				0.602
DEM	0.457	0.436				0.565
ZAK	0.427	0.515				0.540
BUN	0.431		0.410	0.491		
KRU	0.414			0.500	0.436	−0.435
CAP				0.455		

ed in bars 18–20; and a pronounced lengthening of the last IOI in MG6a (position 4–8). The pianists whose interpretations come closest to this pattern are ARR, NEY, BRE, and ASH.

b. Factor II (VAF=17%). This is the "Horowitz factor." It is characterized by significant lengthening of the accented tone in MG3a as well as of the preceding unaccented tone; slight deceleration during MG4a; final lengthening in MG5a; and a *ritardando* during MG6a, followed by a sudden shortening of the last IOI. Some of these tendencies were exaggerated in bars 18–20 relative to bars 2–4. Horowitz's actual performances (especially HO2 and HO3) resemble this pattern, except that the exaggeration in bars 18–20 is much more dramatic. HO3 differs in that bars 2–4 and 18–20 are executed very similarly, both showing dramatic lengthening in positions 4–2 and 4–3.

c. Factor III (VAF=14%). This is the "Cortot factor." It is characterized by a fast start, progressive slowing down during MG4a and MG5a with final lengthening in MG5a, and some shortening of the final IOI in MG6a. Some additional peaks appear in bars 18–20. Cortot's real performances are similar, especially CO1 and CO2, except that the tendencies in bars 18–20 are much more exaggerated. In CO3, there is much lengthening in position 20–2 instead of 20–3 (read 4–2 and 4–3 on the abscissa in Fig. 12). Note the sizeable *negative* loading of ORT.

d. Factor IV (VAF=14%). This factor shows pronounced final lengthening in both MG3a and MG4a, a "flat" MG5a with slight final lengthening, and a smooth *ritardando* through MG6a, with a very long last IOI. Bars 2–4 and 18–20 are executed very similarly. This beautifully regular pattern comes closest to the actual interpretations of KUB, NOV, and SCH.

e. Factor V (VAF=11%). This factor shows a rather flat pattern, with initial acceleration and pronounced final deceleration, but with the final IOI shorter than the penultimate one. This pattern is the least differentiated, as if the whole MG chain were thought of as a single gesture. This pattern comes closest to MOI's performance, though he does show some small local peaks in positions 3–5 and 4–1.

f. Factor VI (VAF=7%). This factor begins with a pattern for MG3a and MG4a that is very nearly the opposite of factor II, MG5a shows striking final lengthening, and MG6a shows only a small lengthening of the final IOI. There is an overall trend to accelerate during the MG chain. This pattern is not representative of any individual performance and occurs only in mixed patterns, such as those of KLI, DEM, and ZAK, all of which exhibit pronounced lengthening in position 4–3.

In summary, the information contained in Fig. 12 and Table VI offers only a moderate amount of data reduction. The timing patterns of individual artists were remarkably diverse. Nevertheless, there are many possible patterns that never occurred, such as lengthening in position 3–4 (the second IOI of MG4a), which clearly was treated as transitional by all pianists. Nor were the patterns random in any way; the majority of the artists produced very similar timing patterns in bars 2–4 and 18–20, and also in the two repeats of bars 2–4 (which were averaged here). The performances with the highest consistency between bars 2–4 and 18–20 were ARR, CAP, HO1, and KUB, whereas the most variable ones were BUN, CO1, CO2, HO2, HO3, and ORT. That Cortot's and Horowitz's within-performance variations were carefully planned is suggested by the fact that they were replicated in different performances (only HO1 retained the pattern of bars 2–4 for bars 18–20). Whether BUN's and ORT's inconsistencies were similarly planned is not known, since only a single performance by each pianist was available for examination.

5. MG3b–MG6b

The type-b version of the MG chain occurs three times in the piece: first in bars 6–8, and then in bars 10–12 and 14–16. Because of the substantial *ritardandi* that occur during MG6b, it was decided to treat the last six IOIs of each chain separately. The remaining $3 \times 12 = 36$ IOIs were subjected to principal components analysis, which yielded six significant factors, just as for the type-a passage. They accounted for 80% of the variance. Their timing profiles are shown in Fig. 13, and the matrix of factor loadings is shown in Table VII. As for the type-a chain, the factor patterns are only rarely representative of individual artists' patterns, which more often are a combination of several factor patterns. It may be recalled that the grand average timing profile showed a fairly regular lengthening of pre-accented and accented IOIs in each MG, which really represents final lengthening of the accompanying secondary MG, which ends with an accented tone. The factor profiles essentially represent a varying focus on individual MGs in the chain.

a. Factor I (VAF=22%). This factor shows substantial

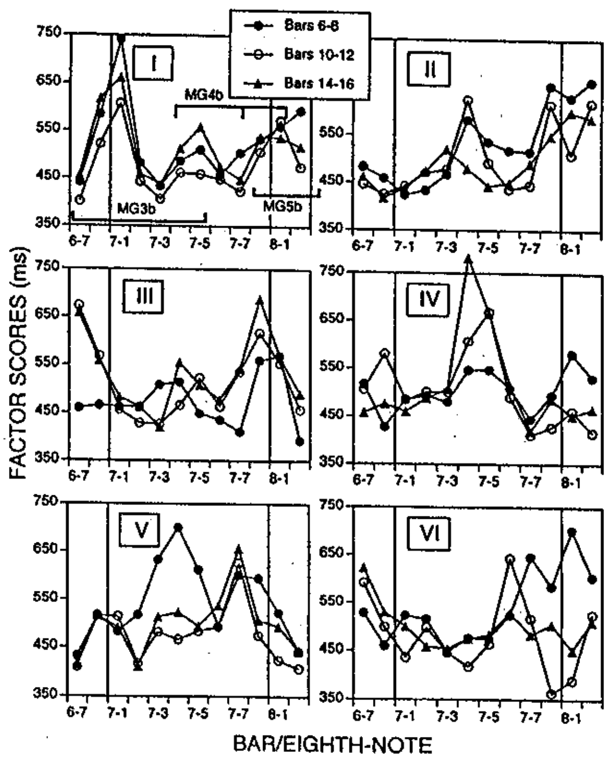

FIG. 13. Rescaled factor scores (underlying timing patterns) for the MG chain of type b in bars 6–8, 10–12, and 14–16. The abscissa labels refer to bars 6–8.

TABLE VII. Sorted rotated factor loadings from the principal components analysis of MG3b–MG5b. (Loadings below 0.4 are omitted.)

	I	II	III	IV	V	VI
ARR	0.923					
HO1	0.788					
ARG	0.745					
HO3	0.740					
KLI	0.715					
MOI	0.701					
ASH	0.648	0.521				
DEM	0.589		0.582			
ESC	0.553			0.490		
GIA	0.531	0.455				
CAP	0.515			0.438	0.450	
CO1		0.860				
KRU		0.829				
BRE		0.733				
CO2		0.699				
NOV	0.418	0.691				
SHE		0.675			0.407	
CUR		0.664	0.488			
SCH		0.629	0.404			
KUB		0.530		0.416		
NEY			0.814			
KAT			0.746			
DAV			0.721			
ZAK	0.407		0.642		0.416	
ORT				0.893		
BUN					0.756	
CO3						0.863
HO2	0.413				0.452	

lengthening in MG3b, much less in MG4b, and somewhat more in MG5b. The timing profiles for the three instances of the MG chain are very similar. The pianist most closely associated with this pattern in ARR, and a number of other artists, including Horowitz, show substantial loadings in this factor. (In constrast to the type-a chain, there was no "Horowitz factor" here.)

b. Factor II (VAF=21%). This factor is characterized by total absence of lengthening in MG3b and variability during MG4b and MG5b, usually with more preaccentuation lengthening than accentuation lengthening. Two performances showing this pattern in relatively pure form are CO1 and KRU, and several other performances (including CO2) show substantial loadings. Thus this was again a kind of "Cortot factor," though it was not unique to Cortot. However, CO3 deviates from this pattern and is responsible for a unique factor, after all (factor VI).

c. Factor III (VAF=14%). This factor is characterized by a flat pattern (bars 6–7) or pronounced *accelerando* (bars 10–11 and 14–15) during MG3b, variable lengthening in MG4b, and pronounced lengthening in MG5b. NEY, KAT, and DAV are the best representatives.

The remaining three factors account for much less variance (8% in the case of factors IV and V, 6% in that of factor VI), and each of them is largely associated with one particular artist. These three artists, which earlier analyses have already revealed to be somewhat eccentric, are ORT, BUN, and CO3. Thus ORT put increasing emphasis on MG4b in successive renditions, BUN showed a very atypical peak in position 7–7 and tremendous lengthening in MG4b in bar 7 only, and CO3 showed no lengthening in MG3b and MG4b, and a very variable execution of MG5b.

On the whole, within-performance variability across the three instances of the type-b chain seemed larger than for the type-a chain, whereas between-performance variability seemed somewhat lower. The former observation is not surprising, for the two MG chains of type a are identical, whereas the three chains of type b differ harmonically and melodically. No pianist showed nearly identical patterns for all three instances of type b. Some pianists were markedly more variable than others, however, not to say erratic. BUN leads the list, which also includes ARG, CO3, DAV, and MOI. The relatively most consistent pianists were ASH, DEM HO2, KLI, NEY, and ZAK. Some pianists (BRE, KRU, SCH) were notable for their relatively flat and uninflected rendering of the whole type-b chain.[13]

6. The *ritardandi*

As is evident from the grand average timing pattern plotted in Fig. 3, major *ritardandi* occurred at the ends of periods and phrases. The largest *ritardando* is in bars 23–24, at the end of the piece. A substantial slowdown occurs also in bar 16, at the end of period B. Less pronounced *ritardandi* occur in bar 8, at the end of period A, and in bar 12, at the end of phrase b2. Thus there are four locations where major *ritardandi* are observed. It may be asked whether the temporal shapes of these *ritardandi* followed any common pattern.

This question was addressed previously by Sundberg and Verrillo (1980) and Kronman and Sundberg (1987) with regard to the *ritardandi* observed at the ends of performances of "motor music," mostly by Bach. Sundberg and Verrillo plotted an average "retard curve" in terms of inverse IOIs and observed that it could be described in terms of two linear functions: a gradual slowdown followed by a more rapid one. The latter phase often included only three data points, however, and tended to coincide with the last melodic gesture in the music. Kronman and Sundberg abandoned the bilinear model and interpreted the average retard curve as a single continuous function. That function was described well by expressing local tempo (1/IOI) as the square root of the normalized distance from a hypothetical zero point located one beat beyond the onset of the final tone. Kronberg and Sundberg speculated that this function may be an allusion to physical deceleration in natural activities such as walking.

That the average *ritardando* followed a quadratic function is intriguing given the present results concerning the temporal shape of MG2. The *decelerando* in that ascending melodic gesture may be considered an instance of a local *ritardando*, and the principles involved may be quite the same. A square-root (convex) function fitted to reciprocal IOIs implies a quadratic (concave) function fitted to the original IOIs. However, it is difficult to reach strong conclusions from Sundberg's examination of timing patterns averaged across performances of different music. The melodic grouping structure must be taken into account, which was done only to a very limited extend by Sundberg and his colleagues.

This is much easier, of course, when dealing with multiple performances of a single piece of music, as in the present case. Inspection of the grand average timing profile in Fig. 3 suggests that *ritardandi* progress smoothly within melodic gestures but are interrupted between gestures. Thus the final *ritardandi* in bars 12 and 16 (read bar 8 on the abscissa) can be seen to begin with the second IOI, but a "reset" occurs after the fourth IOI, which corresponds to a gestural boundary. The final lengthening of MG5b causes the initial *ritardando* to "overshoot" its trajectory, which is resumed with the onset of MG6b. These *ritardandi* thus are divided into two sequences of three and four IOIs, respectively. Similarly, in the large final *ritardando* in bars 23–24, which begins as early as position 23–3 or 23–4, two resets occur, corresponding to the ends of MG4a and MG5a, which in this instance are unmistakably indicated in the score by "commas." This grand *ritardando* thus is divided into three groups of IOIs: 3(4), 4, and 2. Because meaningful fitting of quadratic curves requires at least four data points, the following analyses examine one sequence of four coherent IOIs from each of the three *ritardandi*, forming either part of MG6b or of MG5a. The analyses were analogous to those conducted on MG2.

In each case, it was found that a quadratic curve fit the average IOI pattern quite well, perfectly so in the case of MG5a. These curves are shown in Fig. 14. Quadratic functions were subsequently fitted to all 28 individual performance timing patterns of each *ritardando* section. Most fits ranged from excellent to acceptable (see footnote 9). In the case of MG6b (bar 12), there were five clearly deviant cases (ARG, HO3, KUB, and ORT, all of whom shortened the last IOI, and KLI, who showed a rather flat pattern); for MG6b (bar 16) there was none; for MG6a there was one (ARG, who shortened the last IOI).

The coefficients of the quadratic equations describing the acceptable (and better) fits were found to be highly correlated in all instances, just as for MG2. Thus it was again possible to construct families of parabolas that capture a major portion of the individual variation in the shapes of the *ritardandi*. These curves are shown in Fig. 15. Taking into account the different time scales on the abscissa, it seems that each *ritardando* has its distinctive range of variation, but all show the property of being largely parabolic in shape. These results for strictly intragestural *ritardandi* support the observations on more heterogeneous materials by Sundberg and Verrillo (1980) and Kronman and Sundberg (1987).

A final analysis was conducted on the last two IOIs of the piece. While it did not make sense to fit any function to them, their correlation could be examined, which was 0.87. Thus, the longer the penultimate IOI, the longer the final IOI.[14] Not only was the relationship quite linear, but the regression line passed almost through the origin, suggesting that the two IOIs were in a constant proportion (1:1.8).

V. GENERAL DISCUSSION

The present comprehensive analysis of expressive timing patterns in 28 performances of Schumann's "Träumerei" provides an objective view of the commonalities and differences among great artists' interpretations of one of the masterpieces of the piano literature. At first glance, the differences are perhaps more striking than the commonalities. There is ample material here to support the view that every artist's performance is, in some sense, unique and unlike any other artist's, even if just one physical dimension (timing) is considered. Even the same artist's performances on different occasions, while demonstrably similar, are sufficiently different to be considered distinct and individual events. Yet, there are also significant commonalities that apparently reflect constraints on performance, at least on those performances that have been deemed suitable for commercial distribution. The individual variations among performances largely take place within these constraints, although there are always exceptions, representing conscious or unconscious transgressions of the boundaries established by musical convention.

How should these boundaries be characterized? And are they purely conventional (i.e., arbitrary), or do they represent more general laws of motor behavior and perception that music performance must conform to in order to be naturally expressive?

It is probably futile to attempt to characterize the boundaries of acceptable performance practice. They are manifold and are likely to contract and expand as a function of many factors. It is theoretically more parsimonious to con-

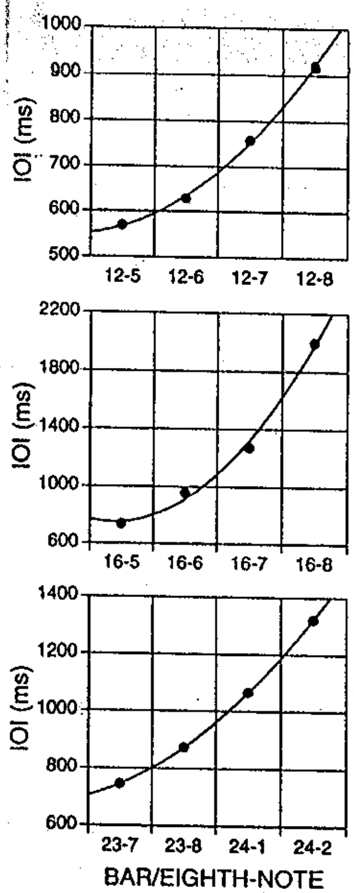

FIG. 14. *Ritardando* functions: Quadratic functions fitted to the grand average (geometric mean) IOIs in MG6b (bars 12 and 16) and in MG5a (bars 23–24).

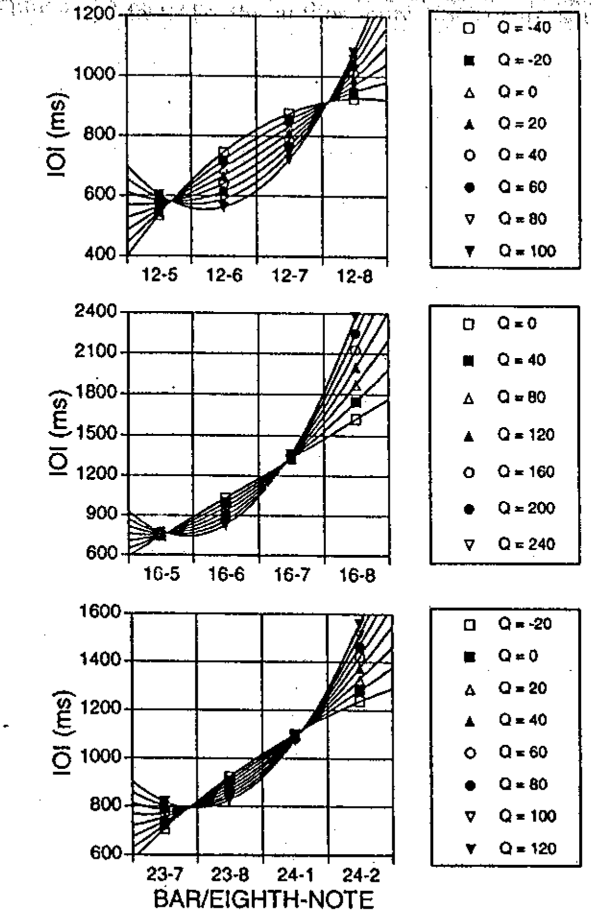

FIG. 15. Families of parabolic timing functions for the *ritardandi* in MG6b (bars 12 and 16) and in MG5a (bars 23–24).

ceive of a performance ideal (norm, prototype) that lies at the center of the hypothetical space enclosed by the boundaries. This ideal may be thought of as a relatively abstract specification that contains free parameters, so that a multiplicity (if not an infinity) of concrete performances can be generated that all more or less satisfy the norm. Because of the enormous complexity of serious music, any concrete performance ideal is very difficult to attain; and, if one were attained, the artist would probably change it the next time, because art thrives on variety. However, the underlying abstract ideal may remain constant, as it embodies generally accepted rules of performance practice. Concrete realizations are conceived, perceived, and judged with reference to the underlying norm, but deviation from the norm (within certain limits of acceptability) can—to some extent, must—be an artistic goal. That is, diversity is as necessary as is commonality: Both uniformity and lack of an aesthetic standard are detrimental to art.

One obvious free parameter is tempo. Even though there may be an "ideal tempo" for a piece of music, this is again an abstraction. There is in fact a *range* of acceptable tempi, and individuals may differ considerably in what they consider "the" ideal tempo. This is illustrated by the performance sample examined here, which represents a very wide range of tempi, nearly all of which seem acceptable to a musical listener (the author must rely on subjective judgment here), even though some sound clearly slow, while others sound fast. To the author, only ESC really sounds "too slow" (though perhaps even that judgment might change if his performance were heard in the context of the whole "Kinderszenen" suite), while that of DAV sounds "too fast," though apparently (Clara) Schumann wanted it that way. An unusually fast "Träumerei" indeed seems to come across better than an unusually slow one.

The present investigation substantiates two abstract timing constraints embodied in the hypothetical performance ideal. One of them concerns the temporal marking of the melodic/rhythmic structure, the other one has to do with the temporal shaping of the individual melodic gestures, particularly of the *ritardandi* within them. The first constraint is by now well known and is implemented in Todd's (1985) model of expressive timing at the phrase level. The principle is that boundaries in the hierarchical grouping structure are generally marked by *ritardandi* whose extent is roughly proportional to the "depth" of the boundary. Thus the most extensive *ritardando* occurs at the end of the piece, where boundaries at all levels coincide; substantial *ritardandi* occur at the ends of major sections, such as 8-bar periods; smaller *ritardandi* occur at the ends of individual phrases and gestures. A performance system such as that devised by Todd may come close to the principles embodied in the performance ideal, though it is not known at present whether a performance strictly following such rules would in fact be perceived as "ideal" by listeners. There is some perceptual evidence, however, that listeners—even those without much musical education—expect phrase-final and gesture-final lengthening to occur (Repp, 1992a).

What is clear from the performances examined here is that variability increases at lower levels of the structural hierarchy. Virtually every performer observes the major *ritardandi* at the ends of major sections, though in different degrees. When the complete performance timing profiles were subjected to principal components analysis, only a single factor emerged, which reflected the qualitative conformity in that regard. When the analysis was restricted to the first 8 bars only, four factors emerged. When the analysis focused on half a phrase, omitting major *ritardandi*, six factors emerged. It seem paradoxical that the number of independent factors increases as the number of data points decreases. What this reflects is the increasing pattern variability at lower levels of the structural hierarchy.

This variability is not likely to reflect a decrease of control over precision in timing, though some of it may. Pianists at the level of accomplishment studied here can control their timing patterns down to a very fine grain. What the variability presumably reflects is a relaxation of the performance constraints imposed by the grouping structure at lower levels of the hierarchy. Not only are the boundaries between individual melodic gestures perhaps less definite that those between larger units, but they are weaker determinants of the timing pattern and compete with other local factors including harmonic progression, melodic pitch contour, and texture. It also seems that the major source of timing variation is not artists' choice of unexpected locations for expressive lengthening (though some of this occurred, too) but varying degrees of emphasis on expected locations. Thus pianists often omitted lengthening where the detailed grouping structure might have predicted it, whereas they overemphasized other grouping boundaries, as if to compensate. Sometimes this resulted in the creation of hypergestures, which combined two or three elementary melodic motifs. In other words, the lowest level of the grouping hierarchy is structurally flexible; it permits the marking of group boundaries but does not prescribe it. Artists choose from among the possibilities in a manner comparable to varying focus in a spoken sentence.

The other constraint observed in the present data is more novel and more tentative. It is that, within melodic gestures requiring a *ritardando* for whatever reason, this local tempo change is best executed such that successive IOIs follow a parabolic function. This constraint was not only observed in the majority of performances at four different locations in the music (nine, if the different occurrences of MG2 are counted separately), but it also is in agreement with the observations of Kronman and Sundberg (1987) on the shape of final *ritardandi*, as originally described by Sundberg and Verrillo (1980). Sundberg and Verrillo also provided perceptual data suggesting that listeners prefer *ritardandi* corresponding to the original (quadratic) timing curves over other possible timing profiles, and a recent study by this author (Repp, 1992b) on the perceptual evaluation of different timing patterns for MG2 led to similar conclusions.

It is not clear at present whether the parabolic timing constraint can also be applied to melodic gestures that do not include any pronounced *ritardando*; certainly other factors, such as accent location, would have to be taken into account. The larger the number of successive tones in a melodic gesture, the stronger the constraint is likely to be; a minimum of five tones (four IOIs) is required. It also remains to be seen whether a similar timing constraint operates at higher levels of the grouping hierarchy. Todd (1985) postulated a parabolic timing function as the prototype for within-group timing at the phrase level, though apparently more for the sake of convenience than for any stringent theoretical or empirical reason. Kronman and Sundberg (1987) hint at a grounding of the parabolic timing constraint in elementary principles of (loco)motion, but without elaborating on this hypothesis. Todd (1992b) claims that a piecewise-linear function is sufficient to describe local timing changes during a performance, but his own (rather limited) data suggest that a quadratic function provides a better fit. At this point, the general hypothesis may be stated that a parabolic timing profile is in some sense "natural" for both performer and listener, and probably not for purely conventional reasons. A better understanding of the origins of this constraint may elucidate the meaning of the motion metaphor that is often applied to music and its performance (cf. Todd, 1992a,b; Truslit, 1938).

The present performance analyses revealed no clustering of individual artists according to sex, age, or national origin. Their individuality apparently transcended these other factors, whose relevance to music performance may be questioned in any case. There is no doubt, however, that in this sample of highly accomplished artists, perhaps precisely because of their level of artistry, some performances seemed unusual and even deviant. Subjective impressions will have to suffice for the time being: Long before objective measurements confirmed the actual deviations in their timing patterns, the performances by Argerich, Bunin, and Cortot, and to a lesser degree those by Horowitz and Ortiz, struck the author as eccentric and distorted. Interestingly, some of these pianists (especially Bunin) also turned out to be inconsistent within their own performance, as if they had no fixed concept and were exploring possibilities. Other artists, however, were remarkably consistent, for example Arrau. The author's subjective impressions were not only highly reliable on relistening, but they also correspond to what professional critics have to say about some of these pianists' performances. Certainly, Horowitz and Cortot are two of the most unusual pianists of this century, and their greatness may lie precisely in their individuality, which challenges the listener. However, in the context of listening to 28 different performances in sequence, which heightens one's sensitivity to differences and perhaps increases one's reliance on an internalized performance ideal, the unusual performances do not fare so well. The author's favorites are the performances by Curzon, Brendel, and Ashkenazy, which usually were in the middle field in the various analyses reported above, and hence were mentioned only rarely. The author's aesthetic ideal seemed to correspond to the central tendency of the sample examined here. Whether this ideal represents a stable representation of traditional performance norms or whether it is a form of psychophysical adaptation to the range of the performance sample is an intriguing question that warrants further investigation. Also, the absence of the context of the preceding and following pieces of "Kinderszenen" must be acknowledged; interpretations that sound unusual in isolation may sound more convincing in context. Finally, it must be noted that the author's impressions derived not only from the timing patterns of these performances, but also from their intensity patterns (both "horizontal" and "vertical"), their articulation and pedaling, and their overall sound quality. There are many parameters that contribute to the subjective impression of a performance, but the timing pattern is probably the most important one. Nevertheless, future performance analyses will have to consider these other parameters (which are much more difficult to measure in recorded performances) as well as the subjective impressions of more than one experienced listener. Ultimately, we would like to know *why* individual performers play the way they do, and what message they are sending to listeners (cf. Kendall and Carterette, 1990).

The present research illustrates one of two complementary approaches to the objective investigation of musical performance. The other approach is represented by the work of Sundberg and his colleagues on a system of performance rules (e.g., Sundberg, 1988; Sundberg *et al.*, 1989; Friberg, 1991) and by Todd's (1985, 1992a,b) modeling of musical motion. These models are ingenious and important, but they need to be tested on large performance data bases, which do not exist at present. In this study, a very modest beginning was made toward accumulating and organizing data of sufficient depth and variety (though still restricted to a single composition and a single parameter, timing) to present a challenge and testing ground for emerging models of music performance.[15] Soon, of course, much more extensive data bases should become available with the help of technological marvels such as the MIDI-controlled grand piano. There are exciting times ahead for research on music performance, one of the most advanced and culturally significant human skills.

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[1]An effort is made in this manuscript to distinguish between *notes*, which are graphic symbols, and *tones*, which are sound events. However, when it comes to grace notes, the distinction cannot easily be made, since "grace tone" is not an acceptable term and "grace-note tone" is awkward. It should be understood, then, that "grace note" refers either to the notated symbol or the resulting sound, depending on the context. The same ambiguity holds for "chord," although "tone cluster" is an acceptable term for the sound event and may be used on occasion.

[2]In a third case (NEY), it was discovered that the two repeats were virtually identical from bar 5 on, suggesting duplication by the recording engineers (see Repp, in press).

[3]In the designation for a "position," such as 5-8, the two numbers stand for the bar and the eighth-note IOI within it, respectively. In an expression such as "bars 5-8," however, the two numbers stand for the first and last bars referred to in the range.

[4]Most of the performances in the latter group strike the author as mannered. It seems that these pianists deliberately tried to play differently from the norm, but were not willing or able to do so consistently. Perhaps they intended to convey an improvisatory quality.

[5]Todd (1992a,b) has recently revised and extended his model of expressive timing. An application of this model to the present data would be most interesting but exceeds the scope of this paper.

[6]That is to say, there may be an implicit, *expressively modulated* eighth-note pulse going through the longer IOIs. As suggested by the extent of the MG boxes in Fig. 2, the first of these pulses "belongs to" the preceding MG, but its actual duration cannot be determined unless it is marked by some tonal event in another voice. MG2i, which breaks up the intergesture interval in the soprano voice, may track the implicit pulse induced by the primary MG2 and thus may reveal its final lengthening.

[7]Although these performances were by no means identical, they were more similar than almost any pair of performances by different artists. Cortot's three performances intercorrelated between 0.80 and 0.82, and Horowitz's between 0.81 and 0.92. Only three other values in the 28×28 intercorrelation matrix exceeded 0.80: CAP/ZAK (0.86), KAT/SHE (0.82), and CUR/KRU (0.81). On the other hand, some of the lowest correlations were observed between Cortot's and Horowitz's performances (0.15 to 0.44), and also between Cortot and several other artists (ARG, BUN, ESC, KLI, NEY, NOV). Only two other correlations fell below 0.30: BUN/MOI (0.26) and HO1/SCH (0.26). Most correlations were between 0.4 and 0.7. Thus Cortot and Horowitz were rather extreme cases in the present sample, which agrees with their general reputation as highly individual artists.

[8]The possible covariation of adjacent as well as distant IOIs was investigated in a principal components analysis on the large (190×190) matrix of intercorrelations among all IOIs, computed across the 28 performances. Its purpose was to "retrieve" the melodic grouping structure through the factors extracted, on the expectation that IOIs would covary more strongly within MGs than between. This expectation was not fulfilled; the analysis yielded a very large number of significant factors, none of which captured much of the variance. They did reflect correlations among some adjacent IOIs and especially between IOIs in structurally analogous positions across phrases of the same type. However, for example, there was no significant relationships between the timing of MG1 and MG2, nor even between the beginning and the end of MG2.

[9]Unfortunately, the software used for curve fitting (DeltaGraph) did not provide a measure of goodness of fit. Therefore, statements about this aspect of the data will remain impressionistic in this paper.

[10]The L coefficient can be understood as follows: If the constant C (without loss of generality) is assumed to be 0, the parabola must pass through the origin, the (0,0) point. Here, L represents the slope of a tangent through the origin. It is zero when the minimum of the parabola is located at the origin. It becomes negative when the minimum of the parabola moves to a positive value along the abscissa, but since the left branch of the parabola must still pass through the origin, this implies a negative value along the ordinate for the minimum.

[11]The correlations varied systematically across the six instances of MG2: The highest correlations (0.91-0.93) were obtained for bars 1-2 and 17-18; lower correlations (0.85-0.86) hold for bars 9-10 and 13-14; and the lowest correlations (both 0.79) were found for bars 5-6 and 21-22. The less tight relationship here must be due to variations in global tempo among the pianists, which may be only weakly related to the degree of tempo modulation in MG2.

[12]For that matter, the *fermata* chord in bar 22, which is clearly notated as requiring simultaneity of all tones, was not played by all pianists in that way: DAV and NOV played a grand *arpeggio*, NEY played a partial arpeggio (two grace notes in the left hand, as in bars 2, 6, and 18), and ASH played only the lowest tone in advance. These variants may have been occasioned by small hands (note that three of the pianists are female), for the chord requires a large span.

[13]One detail skipped on this analysis is the timing of the melody grace note during the last IOI of MG5b (position 8-4). Its onset generally occurred near the middle (40%-60%) of the IOI. A few pianists played it a little earlier (CAP, CO2) or later (ARG, ESC, SCH, HO2, ARR). Two artists (CO3, DAV) omitted it altogether.

[14]In a final display of eccentricity, BUN produced a terminal IOI of nearly 5 s duration, with ORT not far behind.

[15]The data matrix is available from the author.

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