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Phonation from a Continuum Mechanics Point of View

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Some aspects of the continuum mechanics of phonation are described here. Why should we be interested in mechanics, either fluid or solid? Further, why should we be interested in continuum mechanics? Consider the two-mass model as an example of a mechanical model in regards to the first question. This is a mechanical model because the input and output parameters are mechanical parameters and physical laws are used to relate them. The two-mass model illustrates the mechanically important energy flow and stability properties. Other kinds of models, perhaps of the vocal tract or of the larynx itself, are important in other contexts to tell us that a model is possible in complicated situations. For instance, it may be too complicated to explain the control of the vocal tract, or even the larynx, using a physically based model. Here, more abstract control models provide "existence proofs" for lawful behaviour.

The continuum approach to mechanics should be pursued because it can bring us closer to physically measurable quantities than can lumped-element models. Continuum mechanics treats materials, like air, as though their parameters, such as density and velocity, vary continuously in space. The smallest spatial scale studied is much larger than the molecular scale. For instance, the Young's modulus of the epithelium can be measured (Titze and Durham, 1987) and predictions made based on a continuum model, as in certain numerical simulations. It is more difficult to relate the physically measurable quantities to a lumped-element model, like the two-mass model.

Following energy flow and being aware of stability properties in the larynx tells us the nature of the sound and the conditions under which sound is produced. We want to know under what conditions the folds will oscillate, or the instability of the open folds under the influence of air flow. When the folds are vibrating, we want to know how much energy is being absorbed by the folds, and how much of the energy that is left in the air will actually contribute to acoustic output. (Some of the energy travelling through the solid or travelling in the air in a nonacoustic mode can later be radiated as sound.) A knowledge of the instability of the air motion itself will tell us how much energy of air motion becomes predominantly nonacoustic turbulent energy. In the first part of this paper, the issues of energy balance and stability in air will be discussed, supposing that the folds are given sufficient energy to oscillate. The mechanics of the mucousal wave will be considered in the second part of the paper. Again, stability and energy balance are the central issues.

AIR MOVEMENT DURING PHONATION

During phonation the air below the glottis and in the lungs has potential energy. When the folds open, a bit of air leaks out so that some of the potential energy is released as the kinetic energy of air motion. If we consider only the possibility of acoustic motion, then the velocity of air at the glottis at time t will be signaled at the mouth at time $t + l/c$, where l is the length of the vocal tract and c is the speed of sound. Note that it is not the particles of air that leave the glottis at time t that are sensed at the mouth at time $t + l/c$, but rather the events are transmitted down the tube by wave action: bits of air pushing neighbouring bits of air. To conserve mass, there must also be a net change in the density of air as the signal travels down the tube, so that the wave is compressive travelling down the tube. (A rarefaction wave is sent travelling toward the lungs.) The magnitude of the signal that actually reaches the mouth will depend on the waves reflected back toward the glottis. For instance, the signal at the mouth depends on the relative quantity of the potential energy in the lungs compared to the energy of the reflected wave (i.e., on whether the air leaking from the glottis depends greatly on the reflected wave: source-tract interaction.)

However, this is not what happens in the real vocal tract situation. The acoustic equations that describe the situation above are only an approximation to the continuum mechanics of the air in the vocal tract. These equations do not take into account things like rotational motion of the air that can result from the viscous shear stress in the air. This rotational motion does not allow us to make the acoustic approximation right away. The continuum mechanics must be considered in its detail.

Viscous shear stress results in a frictional force. Shear stress results when layers of air move by one another at different velocities. The frictional force acts opposite to the direction of motion. So, if the flow is in one direction, and the velocity is changing only in a perpendicular direction (planar flow), then the velocity, at any time, can be plotted as a function of position in that perpendicular direction. The magnitude of the frictional force opposing the motion is proportional to the curvature of the resulting curve. Because of the shear stress, there is a net torque on any small volume of air and rotational air motion will result.

There can be a great deal of this frictional force in the glottis because the air speed must increase from zero at the epithelium to perhaps as high as 5000 cm/sec in a few millimetres at the center of the glottis. This frictional force can set air into rotational motion, which can be convected into the vocal tract. Further, there is air in the vocal tract near the glottal exit that is not travelling at nearly the speeds of the air in the jet exiting from the glottis, leading to large gradients of velocity and the production of more rotational motion. The rotational air motion is not a part of the acoustic motion of the air, although it is also unsteady. There are other arguments to show the existence of rotational motion of air near the glottis based on the conservation of mass and momentum (McGowan, 1988).

Beyond the loss of energy, the rotational motion would appear to have little consequence to the acoustic output of speech. Indeed, in the first approximation, the propagation of sound is decoupled from the rotational motion (Chu and Kovaszany, 1957). But there is an important consequence, because the unsteady rotational motion can act as a source of sound, particularly when it interacts with solids like the vocal folds. I hope that this provides an example of why we should be concerned about energy balance considerations that can be uncovered using continuum mechanics. Stability of motion is also of concern here. If the rotational air motion is turbulent to some degree, the sound sources due to rotational motion will be noisy. The issue of stability of the oscillating phonatory jet should be addressed experimentally.

MECHANICS OF THE MUCOUSAL WAVE

In this section of the paper I will describe a continuum mechanics approach to the mucousal wave of the vocal folds. This analysis will allow us to consider the growth of the mucousal wave based on the mechanical properties of the biological materials and other clinically important materials. The continuum approach gives a more straightforward mapping from the real larynges to the model than does the lumped element approach. However, the lumped element approach, as exemplified by the two-mass model, can tell us much about the proper approximations to include in the simplest continuum model.

The first hint that the two-mass model gives is that only pressure stress, and not viscous shear stress, need be considered in relation to the force balance on the solid surface. (Viscous shear stress is considered in the calculation of flow resistance only, and as has just been demonstrated, has important consequences in the motion of the air). Another hint is that only the dynamics of the surface of the folds in a direction normal to the mid-line of the glottis need be considered. The forces of air on the solids in the direction of the vocal tract axis are of no consequence as far as the motion of the masses is concerned. (Recall that only the simplest model is being considered.) Therefore, the particular geometric feature of the folds jutting toward the centerline in relation to the solid boundaries of the trachea and pharynx has constriction of flow as its only mechanically salient feature. This constriction results in high air speeds in the glottal region and a resistance to flow caused by the formation of rotational fluid motion at the entrance and exit of the glottis.

All these simplifications will be made here, along with the further restriction that the initial steady air velocity be the same throughout the air channel, although the effects of flow resistance due to rotational air motion at the entrance and exit to the glottis will have to be included. That is, the effect of the glottis here is to obstruct the flow and not to speed it up. This makes the problem tractable, while still illustrating the basic wave growth mechanism. More sophisticated analytical techniques are needed to include this spatial variation in the initial steady air velocity.

To begin the continuum model, Hirano's description of the vocal folds is abstracted for the model layered structure (Hirano, 1981). There is a layer of air flowing with velocity U over a thin, elastic, stiff epithelium, a fluid Reinke's space, and a ligament-muscle impedance boundary. The epithelium is supposed to lie in the $z = 0$ plane and the air flows in the x-direction. The y-direction will be the breadth dimension in this essentially two-dimensional picture. (The $z = 0$ plane will be called the horizontal plane and the z-direction is the vertical direction).

The sequence of steps in this analysis is as follows. The equation of motion for the epithelium will be stated. It is supposed that a spatially sinusoidal perturbation is applied to this layered structure in the x-direction, which will produce pressure perturbations in the air and Reinke's space and vertical position changes in the epithelium. The spatial variations will be related to the time variations through a complex number called wave speed, c. The question to be answered is: Do these perturbations grow or decay in time? The answer to this question will depend on where in the complex plane c lies. If the perturbations grow in time, then the structure is linearly unstable to the particular sinusoidal perturbation and a wave will be presumed to result.

The equation of motion for the model epithelium for small perturbations from the plane $z = 0$ is (Miles, 1956)

$$m\frac{\partial^2\zeta}{\partial t^2} - T\frac{\partial^2\zeta}{\partial x^2} + D\frac{\partial^4\zeta}{\partial x^4} = -\Delta p \tag{1}$$

where ζ is the elevation above $z = 0$, m the mass/area, T the tension/breadth, D the bending stiffness/breadth, $\Delta p = p_{a\mid z=0} - p_{\mid z=0}$, p_a the perturbation air pressure and p the perturbation Reinke's space pressure.

As the air pressure increases relative to the pressure in Reinke's space, the epithelial surface tends to move in the direction of Reinke's space. It is restrained from moving too far by the tension and bending stiffness. Bending stiffness will be neglected for now, as it can be included in a later analysis without much work. The dynamics of the epithelium are described by

$$\frac{\partial^2\zeta}{\partial t^2} - \frac{T}{m}\frac{\partial^2\zeta}{\partial x^2} = -\frac{\Delta p}{m} \tag{2}$$

This appears to be a forced wave equation with the restoring force proportional to the curvature of the epithelium. The stronger the constant of proportionality, that is, the larger T/m, the faster the wave travels, because the wave speed equals the square root of this quantity. Recall that quantities that obey the unforced wave equation travel without change of shape.

The apparent forcing is provided by the difference in the air and Reinke's space pressure at the epithelium. However, it turns out that this difference in pressure is a function of the surface elevation, ζ, so that the differential equation is really an unforced, homogeneous equation. One might want to think of this as a system with very strong feedback. After finding the pressure difference in terms of the elevation, ζ, it will be seen that the equation is no longer the simple wave equation, but a slightly more complicated equation allowing for wave-like behaviour, but where surface shapes can change over time. In fact, some solution shapes may grow in amplitude, so that these shapes can be expected to become visible from very small initial perturbations. These shapes are linearly unstable.

Recall that sinusoidal perturbations are to be considered. First, it is supposed that all quantities can be Fourier decomposed in space and time, so we write

$$f = \tilde{f} e^{ik(ct - x)}, \quad \tilde{f} = \tilde{f}(z) \tag{3}$$

for all dependent variables, where the real part of the above expression is taken. k is a real number known as the wave number and equals $2\pi/\lambda$, where λ= wavelength. By making this substitution, the stability of the surface to sinusoidal perturbations will be studied. c is the wave speed, which may be complex. The real part of c times the wavenumber gives the circular frequency. If c has a negative imaginary part, there is growth in time. One of the goals here is to find the value of c by substituting equation (3) into the equation of motion, equation (2). Before this can be done, the pressure difference, Δp, must be written in terms of the elevation, ζ. That is, the feedback must be found explicitly.

The expressions for air pressure and Reinke's space pressure depend on the governing equations and the boundary conditions at the mid-line of the air channel and at the ligament-muscle boundary. It can be shown that (McGowan, 1989)

$$\tilde{p}_{a\mid z=0} = \frac{\rho_a}{k\tanh(kh_0)}\left\{\frac{\partial^2\tilde{\zeta}}{\partial t^2} + 2U\frac{\partial^2\tilde{\zeta}}{\partial t\,\partial x} + U^2\frac{\partial^2\tilde{\zeta}}{\partial x^2}\right\} \tag{4}$$

where h_0 is the half-width of the air channel and ρ_a is the density of air. Note that for kh_0 small, or surface wavelength much greater than channel half-width, $\tanh(kh_0)$ is small and the effect of the air on the epithelium is greatly enhanced over the situation where the flow is not channelled. For instance, the Bernoulli effect, as represented by the final term in the above equation is enhanced. This will be seen to lead to a much greater likelihood for the growth of surface waves.

It turns out that if Reinke's space is shallow compared to the surface wavelength, then the epithelium senses the ligament-muscle boundary pretty much directly. If this boundary is modelled as a second-order, locally reacting surface with mass per unit area M, spring constant per unit area K, and damping per unit area R, then

$$\tilde{p}\mid_{z=0} = 0 = \frac{1}{kc}\left\{iR - [M(kc) - \frac{K}{kc}]\right\}\frac{\partial^2\tilde{\zeta}}{\partial t^2} \tag{5}$$

Using equations (4) and (5), we can rewrite equation (2):

$$\mu\frac{\partial^2\tilde{\zeta}}{\partial t^2} + \frac{R}{m}\frac{\partial\tilde{\zeta}}{\partial t} + \frac{T}{m}\frac{\partial^2\tilde{\zeta}}{\partial x^2} + \left[\frac{K}{m} - \frac{\rho_a U^2}{m h_0}\right]\tilde{\zeta} = 0 \tag{6}$$

where $\mu = (1 + M/m)$. Here, the first and second terms in curly brackets of equation (4) have been neglected. Equation (6) still describes a type of wave propagation with damping provided by the ligament-muscle boundary and with an altered restoring force. The damping will tend to decrease the amplitude of any disturbance. The restoring force is altered by the presence of the flow and the underlying ligament-muscle tension, as shown by the factor in the square brackets. This alteration makes the wave dispersive. A dispersive wave is one whose wave speed depends upon wavelength, and this is one reason shapes do not propagate without change.

A nondispersive wave results when equation (2) is taken without forcing. To see this the substitution

$$\zeta = \tilde{\zeta} e^{ik(ct - x)} \tag{7}$$

is made into equation (2) without the forcing term. The following constant wave speed is obtained:

$$c=\sqrt{\frac{T}{m}} \tag{8}$$

The result is that shapes propagate without change in this simpler situation. If, on the other hand, the same substitution is made into equation (6), neglecting damping, the following is derived for the wave speed

$$c=\sqrt{\frac{T}{m}+\frac{1}{k^2}\left(\frac{K}{m}-\frac{\rho_a}{mh_0}U^2\right)}=\sqrt{\frac{T}{m}+\left(\frac{\lambda}{2\pi}\right)^2\left(\frac{K}{m}-\frac{\rho_a}{mh_0}U^2\right)} \tag{9}$$

The wave speed depends on the flow velocity, as well as wavelength.

The instability leading to the formation of the mucousal wave has yet to be considered. First, a little background on this kind of problem will be discussed. When air flows over water, waves can sometimes result. When air flows over water the Bernoulli effect makes the air pressure over the surface less than it otherwise would be. In fact, for air speeds high enough the Bernoulli effect will overcome the restoring forces of surface tension and gravity near the surface, and the water is literally sucked into the air. This is known as a Kelvin-Helmholtz instability, and it is an instability of the static type, where the elevation grows exponentially without oscillation. In this situation, the wave speed, c, is purely imaginary. This type of instability does not depend on the damping in the system, because the effective spring constant changes sign and pushes the mass away from the former equilibrium. It was found, however, that the wind velocities necessary to initiate waves with this mechanism are much higher than those that are observed (Miles, 1959).

Another mechanism was sought that would simply overcome any natural damping in the water, rather than counteracting the surface tension and gravitational forces completely. One such mechanism was proposed by H. Jeffreys in a 1925 paper, and is referred to here as the sheltering hypothesis (Jeffreys, 1925). He proposed that the pressure of the air at the water surface on the windward side of a wave crest is greater than on the leeward side. However, if the airflow contains no vorticity, that is, if the air motion is irrotational this can not be the case. So, Jeffreys proposed that flow separation occurs on the wave crests and that vorticity is shed into the air. This sheet of vorticity divides the windward and leeward sides into regions where the pressure heads, $p_T = p_S + \rho_a U^2/2$, are different. While the velocity of the air at the water surface is zero everywhere, the static pressures, p_S, are different on the two sides of the crest because the pressure heads, p_T, are different. Mathematically, Jeffreys added a term to the equation of motion for surface waves on water that gives a force in phase with the surface velocity, so that work is done by air on the water for waves travelling in the direction of the wind. If the amount of work done is large enough to overcome damping, this leads to what is known as a dynamic instability, where exponentially growing oscillations occur.

As can be seen from equations (6) and (9) it is possible to attain static instability for the epithelium in the case that the air speed is high enough and the surface wavelength long enough to diminish the importance of the tension force of the epithelium. Static instability will also be more likely to occur if T and K are made small, say, by relaxing the cricothyroid muscle, and by making the air channel narrower. If the linear analysis applies to different operating points along the closing phase of the glottal cycle, then the percentage of closing phase spent near or at static instability will help to determine voice type. Creaky voice may be distinguished from chest voice in the amount of the glottal

cycle spent near static instability. In fact, a small change in the parameters used by Ishizaka and Matsudaira in their linear stability analysis of the two-mass model would produce a static instability (Ishizaka and Matsudaira, 1972). However, as in the case of wind waves over water, another instability is usually operable, and it too depends on the loss of pressure head due to the formation of rotational air motion and separated flow.

The two-mass model illustrates this instability very nicely, so this analysis can be used to include the head-loss effect in the continuum model (Ishizaka and Matsudaira, 1972). The head-loss mechanism and the consequent resistance to flow depends on the elevations of the masses, providing another feedback. It was shown that the resulting pressure perturbations because of this dependence affected the lower masses (leftward in our coordinate system), but not, particularly, the upper masses. Further, it was found that the air pressure on the lower masses was higher than it otherwise would have been when the lower mass was away from the mid-line of the glottis and upper mass was raised toward the mid-line. This means that an extra term should be added to the differential equation. Instead of saying that the head-loss affects the lower masses, it will be said that it affects a region weighted by the function $W(x)$, which is substantially non-zero and positive only for $-a < x < 0$. And, instead of saying that the effect's magnitude depends on the elevations of the masses, it will be said to depend on the weighted sum of elevations, so the following term should be included in the differential equation (6),

$$\frac{\rho_a U^2}{mh_0}\, W(x)\int_{-\infty}^{\infty}\Gamma(x')\zeta(x')dx' \tag{10}$$

where $\Gamma(x)$ is positive for $-a < x < 0$ and negative for $0 < x < a$ and substantially zero otherwise. Just as in the case of Jeffreys' sheltering hypothesis, it can be shown that this term produces a force in phase with the surface velocity over a complete cycle for certain kinds of disturbances, meaning that energy will be input into the epithelium. When enough energy is input to overcome the damping, dynamic instability results. Suppose :

$$W(x)=\begin{cases}1 & \text{for } -a<x<0\\ 0 & \textit{otherwise}\end{cases} \qquad \Gamma(x)=\begin{cases}-\sin\left(\frac{2\pi}{2a}x\right) & \text{for } -a<x<a\\ 0 & \textit{otherwise}\end{cases} \tag{11}$$

and make the problem periodic in space by giving W and Γ periodic extensions of period $4a$. Then the problem can be solved by writing ζ in a Fourier series in space, so that each Fourier coefficient is a function of time. Assuming that only the forward going wave of wavelength $4a$ is present, the following differential equation for that coefficient can be derived:

$$\frac{d^2\overline{\zeta_1}}{dt^2}+\frac{R}{m\mu}\frac{d\overline{\zeta_1}}{dt}+\left[\frac{T}{m\mu}k_1^2+\left(\frac{K}{m\mu}-\frac{\mu_B}{\mu}\right)\right]\overline{\zeta_1}=\frac{\mu_B}{\mu}\frac{8}{3}(1-i)\overline{\zeta_1} \tag{12}$$

where $k_1=\frac{2\pi}{4a}$, $\mu_B=\frac{\rho_a U^2}{mh_0}$ and $\overline{\zeta_1}$ = Fourier coefficient for wavelength $4a$ traveling in the $+x$ direction.

Note there is a forcing $\pi/2$ out of phase with the restoring force, called here the out-of-phase forcing. This is provided by the extra term added to the differential equation, and is analogous, although not completely equivalent to the sheltering term used by Jeffreys. The Routh-Hurwitz criterion gives the following inequality for instability

$$\left(\frac{8}{3}\frac{\mu_B}{\mu}\right)^2 - \left(\frac{R}{m\mu}\right)^2\left[\frac{T}{m\mu}k_1^2 + \left(\frac{K}{m\mu} - \frac{11}{3}\frac{\mu_B}{\mu}\right)\right] > 0 \tag{13}$$

The square of the out-of-phase forcing providing the first term in the inequality must be large compared to latter term, which is the square product of damping and effective natural frequency. The natural frequency appears here because the rate of dissipation over a cycle depends on this frequency. Not only does the added head-loss term help to create the instability by increasing the magnitude of the out-of-phase forcing, but it also decreases the natural frequency, and, thus the efficacy of damping.

When the tension T is increased by either tensing the cricothyroid, or a lessening of K, the tension for shorter wavelengths increases. So, it may turn out that there is not sufficient air velocity to create a dynamic instability at the wavelength $4a$, but only at longer wavelengths. However, the instability mechanism depends critically on the relative phase of the epithelial surface at $x = -a$ compared to $x = a$. If the wavelength gets too long then this mechanism will no longer work. So, it appears there must be a critical tension, T, where instability will no longer ensue, at least with this head-loss mechanism. Then such influences as inertial loading or stall flutter must be relied on for instability when phase differences are no longer possible. This is the regime of falsetto voice.

ACKNOWLEDGEMENTS

This work was supported by NIH grants NS-13870, HD-1994, and NS-13617 to Haskins Laboratories.

REFERENCES

Chu, B. -T. and Kovaszany, L. S. G. (1957). Non-linear interactions in a viscous, heat- conducting, compressible gas. *J. Fluid Mech.*, 3:494- 514.

Hirano, M. (1981). *The Clinical Examination of Voice*. Springer-Verlag, Vienna.

Ishizaka, K. and Matsudaira, M. (1972). Fluid Mechanical Considerations of Vocal Cord Vibration, *SCRL monograph #8*. (Speech Communications Research Laboratory Inc., Santa Barbara, Calif).

Jeffreys, H. (1925). On the formation of water waves by wind. *Proc. Royal Soc. London* 107:189-206.

McGowan, R. S. (1988). An aeroacoustic approach to phonation. *J. Acoust. Soc. Am.* 88:696- 704.

McGowan, R.S. (1989). Growth of surface waves with application to the mucosal wave of the vocal folds. *Haskins Laboratories Status Report on Speech Research*, SR 97/98:41-50.

Miles, J. W. (1956). On the aerodynamic instability of thin panels. *J. Aeronautical Sciences* 23:771-780.

Miles, J. W. (1959). On the generation of surface waves by shear flow: Part 3. Kelvin-Helmholtz instability. *J. Fluid Mech.*, 6:583-598.

Titze, I. R. and Durham, P. L. (1987). Passive mechanisms influencing fundamental frequency control. In: *Laryngeal Function in Phonation and Respiration*, edited by T. Baer, C. Sasaki, and K. Harris, pp. 304-319. College-Hill Press, Boston.