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Fluctuations and Phase Symmetry in Coordinated Rhythmic Movements

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Pendular, clocking movements typify mammalian terrestrial locomotion. They can be investigated with a procedure in which people swing hand-held pendulums at the wrists, comfortably and rhyth-mically. Pendular, clocking behavior was examined for in-phase and out-of-phase coordinations. The periodic timing and powering of rhythmic movements in the comfort state follow from different laws (Kugler & Turvey, 1986). One law guides the assembling of the reference frame for "clocking." Another law guides the assembling of the muscular, escapement processes determining the cycle energy. Wing and Kristofferson's (1973) method for parsing periodic-timing variance into independent "clock" and "motor" sources was applied. Mean periodicity was unaffected by phase. "Clock" fluctuations, however, were larger out of phase than in phase. "Motor" fluctuations were indifferent to phase but reflected the departures of individual wrist-pendulum systems from their preferred periods. It appears that an intended phase relation is realized as a constraint on "clock" states. These states are more stable under the in-phase constraint than under the out-of-phase constraint.

Terrestrial locomotion of two-legged and four-legged mammals is characterized by a pendular, clocking mode of organization. Individual limbs are raised and lowered with respect to gravity's pull (the pendular aspect), and when gait and speed are sustained for any interval, the periodic timing of a limb's successive step cycles is closely similar (the clocking aspect). In locomotion, limbs of the same girdle (shoulder, pelvic) swing forward and backward together, or one limb swings forward as the other limb swings backward. The former coordination of the limbs is variously referred to as in phase, 0° difference, homologous, symmetric. The latter coordination of the limbs is variously referred to as out of phase, 180° difference, nonhomologous, asymmetric. Of the three major quadruped gait divisions (Pennycuick, 1975), the walk and the trot engage out-of-phase coordination, the canter (and its fast variants, for example, the gallop) engages in-phase coordination.

The two types of phase relation can be seen in other activities, for example, finger, hand, and arm movements of humans and fin movements of fish. In the present article we investigate claims (Haken, Kelso, & Bunz, 1985; Kelso & Scholz, 1985; von Holst, 1935/1973) that the two phase relations are not movement organizations of like kind, that the in-phase relation is more stable or more "attractive." The analysis presented is in the spirit of an emerging physical biology that emphasizes that the characteristic phenomena of living things are shaped by a small set of strategies common to all of nature's scales (e.g.,

Haken, 1983; Iberall, 1977–1978; Yates, in press). A physical biology chooses physical ideas rather than those of formal systems as the basis of a global theory of the nervous system. Not any set of physical ideas will do, however. It is conjectured that only a variant of statistical mechanics, namely, statistical thermodynamics combined with nonlinear mechanics, has the features required for theorizing about the nervous system in a physical sense (see Yates, 1980).

Rhythmic Behavior of Single and Coupled Wrist-Pendulum Systems

To investigate the dynamics of coordinated rhythmic movements in the pendular clocking mode, Kugler and Turvey (1986) had human subjects swing at the wrists two hand-held pendulums that could be varied in mass and length. The upper panel of Figure 1 depicts the experimental situation. The task of the subject, for a given pair of wrist-pendulum systems, was to swing them together at a comfortable, common tempo. The comfort requirement is a steady-state condition defined over mechanical and thermodynamical variables (see Kugler & Turvey, 1986). The common tempo requirement is an isochronous coupling constraint; the clocking or periodic timing of the two wrist-pendulum systems must be (nearly) identical.

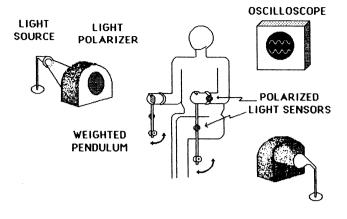
The lower panel of Figure 1 depicts the single wrist-pendulum task. This task yields the preferred periodic timings or natural periods of the individual left and right wrist-pendulum systems. An important question is how the common periodic timing of a left system and a right system, when coupled, compares with the left's natural period and the right's natural period. The simplest expectation is that where the natural periods of the two systems are identical (or nearly so), the coupled period should be of the same magnitude (or very nearly so); and where the natural periods are dissimilar, the coupled period should fall somewhere in between. This expectation follows from assuming that the obvious strategy with systems of unequal natural peri-

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COUPLED WRIST-PENDULUM SYSTEM TASK



Single Wrist-Pendulum System Task

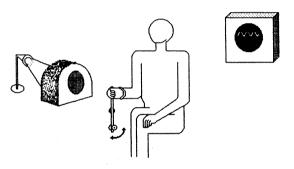


Figure 1. An experimental task for investigating the pendular, clocking mode of rhythmic movement organization in humans.

ods is to compromise by slowing down the naturally faster system and speeding up the naturally slower system. Slowing down and speeding up relative to natural period requires muscular effort and energy expenditure in excess of that for producing rhythmic movement at the natural period. Presumably, at some period bracketed by the left's natural period and the right's natural period there is a minimization of effort and expenditure.

The data obtained by Kugler and Turvey (1986) did not concur with the expectation. Here are two examples from their experiments of the coupling of nearly identical systems: a left system and a right system with natural periods of 791 ms and 787 ms, respectively, coupled at 882 ms; a left system and a right system with natural periods of 911 ms and 892 ms, respectively, coupled at 1,029 ms. And here are two examples of the coupling of very dissimilar systems: a left system and a right system with natural periods of 780 ms and 1,321 ms, respectively, coupled at 1,235 ms; a left system and a right system with natural periods of 864 ms and 1,321 ms, respectively, coupled at 1,188 ms. The obvious account (sketched above) of how two wrist-pendulum systems are coupled to a common tempo is strained by these examples. Why should two wrist-pendulum systems of almost identical natural period settle at a common, coupled tempo that is substantially larger than their natural period? And with respect to the examples of the very dissimilar systems, why should the "compromise" period—the in-between period at which they settle-be larger for the coupling of 1,321 ms with 780 ms than for the coupling of 1,321 ms with 864 ms? Perhaps the coupled period is not determined, as the obvious account suggests, by a strategy that is conditioned by the natural periods of the component systems.

Concept of a Virtual Single System

Kugler and Turvey (1986) showed that the periodic timing of two coupled wrist-pendulum systems is understandable as the periodic timing of a single wrist-pendulum system. It appears that, when coupled, two wrist-pendulum systems defined by six masses (those of the two pendulum rods, two hands, and two sets of weights added to the pendulum rods), oscillating at different distances from two points of rotation, submit to a simpler description—that of a single wrist-pendulum system consisting of a single concentration of mass at a single distance from a single point of rotation. The virtual single length is derived from the equation that converts a compound pendulum into an equivalent simple pendulum (Huygens' law). The equation assumes that a compound pendulum is an ensemble of many "micro" simple pendulums that are (a) of different lengths and (b) rigidly connected. It replaces the multiple "micro" pendulums by a single "macro" pendulum. Figure 2 depicts the transformation. The length of the "macro" pendulum is such that the "macro" pendulum's cycle energy is equivalent to the summed cycle energies of the many "micro" pendulums (Mach, 1893/1960). The many-to-single mapping, the microto-macro conversion, is governed by the conservations of energy and momentum. A conservation—that of mass—similarly governs the determination of the virtual mass of the virtual single system. It is simply the sum of the masses of the two wrist-pendulum systems.

Conservations or summational invariants, it will be recalled. are quantities that are neither created nor destroyed in the course of binary interactions, only redistributed. Most important for present purposes, the conservations completely define the statistical properties of a closed system, that is, the statistical distribution of its subsystems and, therefore, the mean values of the quantities relating to them (Landau & Lifshitz, 1980). Herein lies the basis for the fundamental strategy of statistical physics: The mandatory sharing of the conserved quantities among a body's many subsystems (particles) means that a very detailed description of the body (for example, the motions of each of its subsystems in three dimensions) can be replaced, over some range of conditions, by a very simple description (for example, a single magnitude). Figure 3 depicts the conservation-based conversion of a pair of wrist-pendulum systems into a virtual single wrist-pendulum system. (The attached Appendix gives the details of the calculations used to derive the virtual values.)

The appropriateness of the conversion raises challenging questions about how the nervous system, which is essentially fluid and dissipative, can simulate a system that is rigid and conservative. (Huygens' law is formulated for systems of the latter kind.) Understanding this simulation—the formation of a virtual single wrist-pendulum system—depends, we believe, on an understanding of three key concepts: (a) the universal role of the conservations in assembling unitary processes; (b) haptic (neural) fields as information about kinetic (force) fields; and (c) intentions as boundary conditions or constraints on lawful

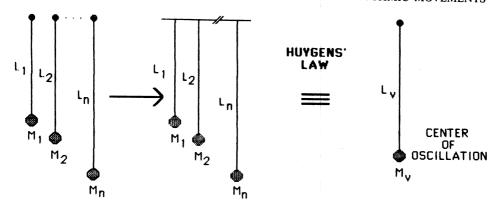


Figure 2. A compound pendulum can be thought of as composed of many point masses at different distances from the axis of rotation. The conception is of many simple pendulums. If they were allowed to oscillate freely (as in the left panel) they would do so at different periods. If they were rigidly connected (as in the center panel), they would oscillate at a single common period. The ratio of the sum of the moments of inertia $(\Sigma m_i l_i^2)$ to the sum of the statical moments $(\Sigma m_i l_i)$ gives the length of the equivalent single simple pendulum (as in the right figure). The subscript v denotes "virtual." The ratio giving the equivalent simple pendulum length is referred to here as Huygens' law.

processes. Each of these notions is touched upon briefly in the present article. A more complete discussion is to be found in Kugler and Turvey (1986).

A Scaling Relation for Wrist-Pendulum Mean Periodic Timing

It follows from the preceding that, given either a pair of wrist-pendulum systems or a single wrist-pendulum system, the task of the subject is very much the same, namely, to assign a single periodic time to a given single length/single mass combination. Using the mammalian tissue constants of density, elasticity, and strength (Economos, 1982), and the tools of similarity analysis and dimensional analysis, Kugler and Turvey (1986) determined that the scaling relation for the wrist-pendulum situation was

$$T_n \propto M^{.0625} L^{.5}, \tag{1}$$

where T_n is natural periodic time, M is the wrist-pendulum mass, and L is the wrist-pendulum length, where M and L are either actual single or virtual single quantities. (It is to be noted that scaling relations, unlike laws, do not have to be dimensionally homogeneous. It should also be noted that wrist-pendular periodic timing depends on both mass and length. A wrist-pendulum system is a hybrid mass-spring/simple pendulum system. There are two potentials at work—the gravitational potential and another elastic potential defined over the neuromuscular system. The mechanical image is of a single pendulum—a mass at one end of a massless rod—oscillating about a fixed point of rotation with a horizontally aligned spring attached to the rod just below the rotation point.)

Given the concept of virtual single system and Scaling Relation 1, we are now in a position to answer the two questions raised above concerning the periodic times resulting from coupling identical wrist-pendulum systems and coupling different wrist-pendulum systems. Let μ stand for $M^{.0625}L^{.5}$, where M is measured in kilograms, and L is measured in meters. The left and right wrist-pendulum systems whose natural periods were 787 ms and 791 ms had the same μ value of 0.422. The μ of the

virtual single system produced by their coupling, however, was 0.446. Similarly, the left and right wrist-pendulum systems that had natural periods of 892 ms and 911 ms had the same μ value of 0.4931; the μ of the virtual single system produced by their coupling, however, was 0.514. By Scaling Relation 1, and with other things being equal, it would be expected that in both of the preceding two instances the periodic timing (882 ms and 1,029 ms, respectively) of the coupled system, that is, the virtual single system, would exceed the preferred periodic timing of its components. Turning to the examples of coupled dissimilar systems, we find that the left and right systems of natural periods 780 ms and 1,321 ms yielded, when coupled, a virtual single system with a μ of 0.723. In comparison, the virtual single system produced by coupling the systems with natural periods of 864 ms and 1,321 ms had a μ of 0.703. By Scaling Relation 1, and with other things being equal, the coupled periodic time in the former case (1,235 ms) should exceed the coupled periodic time in the latter case (1,188 ms).

Indifference of the Mean Periodic Timing Scaling Relation to Phase Symmetry

The pendular clocking mode depicted in Figure 1 differs from quadruped locomotion in that frictional contact with the

¹ The periodic times of left and right wrist-pendulum systems can differ even though the μ value is the same for both systems. Similarly, the periodic time of a left wrist-pendulum system of one subject and the periodic time of a left wrist-pendulum of another subject can differ even though the μ values of the two systems are identical. The reason for these differences is that although steady-state periodic timing is lawfully scaled, lawful scaling does not come neat. The quantities related by Scaling Relation 1 are related in a context. The context is a "coordinate space," a physically based geometry defined by the momentary and local distribution of conserved quantities. The coordinate spaces of the left and right wrist-pendulum activities of a single person need not be identical in the same way that the coordinate spaces of left wrist-pendular activity in two people need not be identical. The abstract notion of coordinate space and its implications for understanding lawful regularities at the scale of biology figure prominently in Kugler and Turvey (1986).

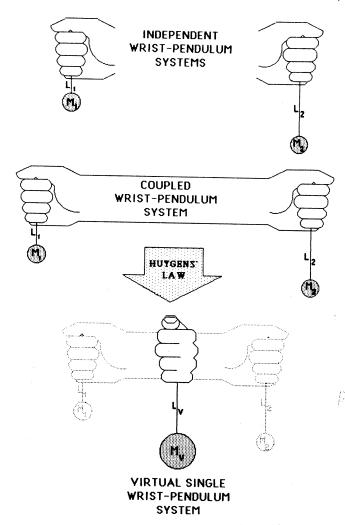


Figure 3. Applying the "compound-pendulum" to "simple-pendulum" transformation (Huygens' law) to a coupled pair of wrist-pendulum systems. (In the resultant virtual single wrist-pendulum system it is as if all the mass $M_{\rm v}$ is concentrated at a single point a distance $L_{\rm v}$ from a virtual point of rotation.)

ground and forward propulsion of the body are absent. Most important, for limbs $M \propto L^4$. Consequently, for quadruped locomotion the relevant scaling relation (see Kugler and Turvey, 1986) is

$$T_n \propto M^{.125} L^0 \text{ or } T_n \propto M^0 L^{.5}.$$
 (2)

Most quadrupeds can locomote in all three major gaits. From one gait to the next, Scaling Relation 2—between periodic time and representative mass and length measures—is invariant (Kugler & Turvey, 1986; Pennycuick, 1975). That is, Scaling Relation 2 holds whether the coupling between limbs of the same girdle is in phase or out of phase. In one experiment, Kugler and Turvey (1986) inquired whether this was also true for Scaling Relation 1. (They expected that it would be given that Scaling Relations 1 and 2 are variants of the same underlying principles linking time, mass, and length in biological systems. The pendular clocking mode of organization in locomotion and swinging wrist-pendulum systems is the same but for a

difference in the mass-length relation.) The question was posed with six pairs of wrist-pendulum systems. For three of the pairs, the μ value of the left wrist-pendulum system was the same as the μ value of the right wrist-pendulum system, and for three of the pairs it was different. The comparison, roughly, is between locomoting with limbs of the same size and locomoting with limbs of different sizes. The six pairings generated six different virtual μ values. For future reference μ_v will designate the μ value of a virtual single system, and μ_l and μ_r will designate the μ values of the individual left and right systems, respectively. In the course of the experiment each of four subjects swung each pair of wrist-pendulum systems at a comfortable, common tempo eight times in phase and eight times out of phase. (The details of the experiment are given below in the Method section.) The main observation was that for each subject the six periodic times related to the six μ_v values in the same way for both in-phase and out-of-phase couplings. Figure, 4 shows the in-phase and out-of-phase periodic times for the six coupled systems, and Figure 5 shows the relation of Log T_n to Log μ_v for the two phase relations (with T_n and μ_v values averaged over subjects). As can be seen, the coefficients (a) and exponents (c) of the two phase relations were nearly identical, and for both phase relations the two quantities, T_n and μ_v , were highly correlated. In sum, as with Scaling Relation 2, Scaling Relation 1 is invariant over the kind of coordination (in phase, out of phase) between limbs of the same girdle.

Three other aspects of the data should be noted. First, the requirement of isochrony (both left and right wrist-pendulum systems clocking with the same period) was met equally by all six coupled systems, and it was met equally in phase and out of phase. The relation of isochrony to system and phase for each subject is depicted in Figure 6. Second, for à given coupled system its period in phase was not consistently longer or shorter than its period out of phase (see Figure 4). When we average the data over the six systems, we find that Subjects 1 and 2 produced numerically shorter periods out of phase than in phase (925 ms vs. 939 ms and 1,015 ms vs. 1,024 ms, respectively) while Subjects 3 and 4 produced numerically shorter periods in phase than out of phase (958 ms vs. 995 ms and 1,034 ms vs. 1,042 ms, respectively). The overall mean difference between in phase and out of phase amounted to only 8 ms (in phase = 1,034 ms, out of phase = 1,042 ms). This difference was not statistically significant, F(1, 3) = 1.08, $MS_e = .407$, p > .05.

The third additional aspect to be noted is that the coupled systems whose components were different $(\mu_l \neq \mu_r)$, that is Systems 4, 5, and 6, were continuous in the Log $T_n \times \text{Log } \mu_v$ plot with the coupled systems whose components were the same $(\mu_l = \mu_r)$, that is, Systems 1, 2, and 3. A useful way to compare coupled, or virtual single, wrist-pendulum systems is through a Symmetry metric (S), where

$$S = (\mu_l - \mu_r)/\mu_v. \tag{3}$$

For the six coupled systems, the s values averaged over subjects were 0, 0, 0, 0.258, 0.378, and -0.457, respectively. The S metric reflects the competing temporal preferences of the individual left and right wrist-pendulum systems relative to the temporal preference of the virtual single system. It is an approximate measure of the compatibility between the steady state of the "macro" virtual single system and the steady states of its constituent "micro" subsystems. Thus, Scaling Relation 1 applies

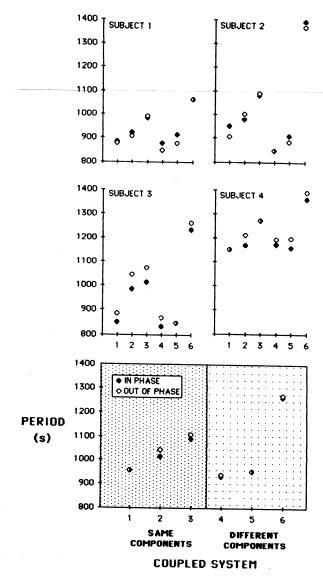


Figure 4. Periodic timing of six coupled wrist-pendulum systems as a function of the intended phase relation. (The lower panel gives the mean data.)

indifferently to the make-up or symmetry of coupled wrist-pendulum systems (see Kugler & Turvey, 1986, for more extensive support of this observation).

Collectively, the observations reported above suggest that there is little to distinguish the pendular clocking mode of organization assembled (a) over in-phase components and over out-of-phase components and (b) over components of the same magnitude and over components of different magnitudes. On the positive side, these observations indicate that left and right wrist-pendulum systems, coupled so as to clock isochronously in the comfort regime, define a unitary process. They point to a soft assemblage and temporary sustainment of a single functional entity—or coordinative structure (Kelso, Tuller, Vatikiotis-Bateson, & Fowler, 1984; Kugler, Kelso, & Turvey, 1980; Turvey, Shaw, & Mace, 1978). This unitizing, or chunking, is primarily the result of the conservational principles effecting

the transformations depicted in Figures 1 and 2. Given the multiple mass/length quantities locally (the left and right wrist-pendulum systems), the conservations condense out single mass/length quantities globally (the virtual wrist-pendulum system). Whether the local components move in or out of phase and whether they are of the same or different magnitudes does not enter into the determination of these global single quantities to which the coupled periodicity is lawfully linked through Scaling Relation 1.

Composition and Fluctuations

This principled insensitivity of coupled wrist-pendulum systems to phase symmetry and composition is with respect to mean periodic timing. In their experimental analysis Kugler and Turvey did not evaluate whether this insensitivity extended to other aspects of the behavior of coupled wrist-pendulum systems, specifically, fluctuations in periodic timing, mean phase relation relative to intended phase, and fluctuations in phase. These other aspects are analyzed in the present article. In the immediately following paragraphs we outline reasons for expecting fluctuational differences where there are compositional differences.

A virtual single wrist-pendulum system—and, by generalization, any coordinative structure (see Kelso & Scholz, 1985; Kugler & Turvey, 1986)—can be likened to a physical cooperativity. In general terms, this is a coherent, unitary state of affairs of an ensemble that is maintained from below by the activity of the atomisms of the ensemble and from above by the field boundary conditions (Iberall & Soodak, in press). The "atomisms" in the case of a virtual single wrist-pendulum system are the left and right wrist-pendulum systems. The "field boundary conditions" are the subject's intentions to swing the pendulums isochronously, comfortably, and at a particular phase relation. A cooperativity is demanding. It is a steady state at the level between field boundary conditions and atomisms that is achieved at the expense of equilibrium at the lower atomistic level (Prigogine, 1980). In natural systems, nonequilibrium at the atomistic level is a source of order at the next, higher level. Let us consider a pair of wrist-pendulum systems more closely. Both systems have a preferred period in isolation (a natural pe-

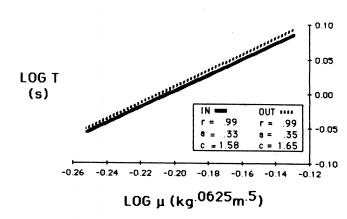


Figure 5. Log (mean) period by log (mean) mass 0625 length $^{-5}$ as a function of in-phase and out-of-phase coordinations. (r = correlation coefficient; a = intercept; c = slope.)

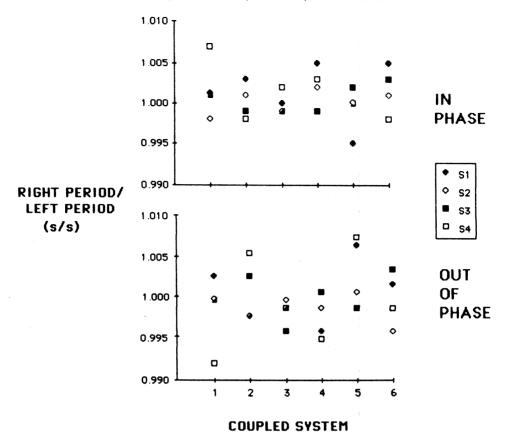


Figure 6. Isochrony of coupled wrist-pendulum systems as a function of subject, coupled system, and intended phase. (If the left and right wrist-pendulum systems were perfectly isochronous, the ratio of right period to left period would be 1.0.)

riod). For the left system the natural period is proportional to μ_l ; for the right system it is proportional to μ_r . These natural periods define time-dependent steady states. When the left and right systems are coupled by common "boundary conditions," the virtual moment variable μ_v will most likely differ from both μ_l and μ_r . This means that the stable coupled period, proportional to μ_v , will not concur with either of the local (left, right) preferences. Consequently, the resultant coupled period can be achieved only by the left and right systems departing from their preferred periods, that is, their steady states.

The S-metric $(\mu_I/\mu_v - \mu_r/\mu_v)$ introduced above provides an index of the degree to which the atomistic level (composed of the individual left and right wrist-pendulum systems) departs from steady state. In natural systems, such departures are ordinarily accompanied by increasing fluctuations that are proportional (up to a limit) to the magnitude of the departure. Consequently, S and fluctuations of the coupled periodic time (the variance s_T^2) should be positively correlated. This is another way of saying that the same/different composition distinction should prove relevant to the behavior of coupled or virtual single systems when the variances of the mean periods, rather than the mean periods themselves, are the subjects of analysis.

Phase Symmetry and Fluctuations

Would the fluctuations in coupled periodic timing be sensitive, similarly, to the in-phase/out-of-phase distinction? They

would if the two phase relations were not equally attractive (that is, not equally stable). The early work of von Holst (1935/1973) on coupled rhythmic fin movements of Labrus and the recent work of Kelso (Haken et al., 1985; Kelso, 1984; Kelso & Scholz, 1985) on coupled rhythmic finger and hand movements of humans points to such an inequality, von Holst (1935/1973) detected the difference in attractiveness through a consideration of the superimposition effect relative to phase. Superimposition refers to the influence of the amplitude of one fin rhythm generator on the amplitude(s) of the other(s) to which it is coupled. Where one fin rhythm generator is dominant, the amplitude of the dominated fin rhythm generator varies with phase. When coupled, the fins of the Labrus oscillate at either one of two phase relations, 180° apart. In one phase relation, the dominant amplitude is added to the nondominant amplitude. In the other phase relation, the dominant amplitude is subtracted from the nondominant amplitude. The coupled fins gravitate to the phase relation at which amplitudes add. The fish that von Holst (1935/1973) studied swim with the main body axis immobile. The pectoral fins of these fish dominate the dorsal fin. Dorsal fin amplitude is larger when the dorsal fin's tip displaces towards the body as the pectoral fin's tip displaces away from the body than when the tips of both fins displace towards the body. Because of the different alignments of these fins to the axis of the body of the fish, the foregoing contrast in directions of coupled displacements amounts to a contrast between in-phase and outof-phase coordinations. By von Holst's analysis, in phase is more attractive than out of phase.

Kelso (Haken et al., 1985; Kelso, 1984; Kelso & Scholz, 1985) reached this same conclusion through an examination of the transition from one phase relation to the other. In the basic experiment, human subjects oscillate the index fingers of the two hands, or the two hands themselves, in the out-of-phase coordination. Initially, the coupled oscillations are at a comfortable period. As the period is continually decreased (the oscillations get faster), a point is reached at which the coordination switches spontaneously to in-phase. Beyond this transition point only the in-phase relation can be sustained. And continuously increasing the period (slowing the oscillations), so as to pass through the transition point in the opposite direction, does not result in a spontaneous return to the out-of-phase coordination. Haken et al. (1985) modeled this "phase transition" phenomenon with a potential function whose local minima correspond to the in-phase and out-of-phase coordinations. In the model equations for this potential function, the out-of-phase minimum exceeds the in-phase minimum. Moreover, the outof-phase local minimum is annihilated at a critical periodicity. As further support for the greater attractiveness of the in-phase coordination, Kelso and Scholz (1985) report that the spectral content of relative phase for out-of-phase coordination becomes biased, at the transition point, to the frequency band characteristic of the in-phase coordination. A 0.8-Hz component is strongly present in in-phase coordination at all periods. This harmonic becomes increasingly prominent in the power spectrum of out-of-phase coordination as the coupled period reduces and approaches the critical period.

To reiterate, in the present article the in-phase/out-of-phase data of Kugler and Turvey are analyzed with respect to fluctuations in period and phase. On the basis of the foregoing observations, it is expected that both the composition of a virtual single wrist-pendulum system and the phasic relation between the components of a virtual single wrist-pendulum system will have consequences for the system's behavior around its mean periodic timing. The analysis is performed, in part, with the help of Wing and Kristofferson's (1973) theory, which partitions the periodic timing variance of rhythmic, biological movements into two components of independent origin.

Independent Bases of "Clock" and "Motor" Fluctuations

In the ordinary man-made pendular clock there is a dissociation of energy and period. The period of the clock is fixed by the mechanical variables of the pendulum—its mass and length. The energy injected into a pendular cycle, to offset frictional losses and to sustain the periodic motion, is fixed by the design of the escapement. An escapement is any process for gating the flow of energy from a potential energy source to a mechanical, kinetic energy sink. In man-made clocks, escapements are impulsive (they inject energy in squirts). In biological clocks, escapements are more often continuous than discrete (Kugler & Turvey, 1986).

Any accidental changes in the magnitudes of the pendulum of an artifactual pendulum clock will not be accompanied by changes in the quantity of energy injected per cycle. Kugler and Turvey's (1986) theoretical analysis of biological pendular clocks in the comfort regime similarly distinguishes between the basis for the preferred periodic time per se and the basis for the escapement energy through which the preferred periodicity is realized. These bases are lawful relations of approximately the form

$$T_n = a(\mu)^c \tag{4}$$

and

$$E = H(f - f_{int}), (5)$$

respectively. Lawful Relation 4 is a fuller form of Scaling Relation 1; if the coordinate space (see Footnote 1) that embeds the relation between T_n and μ is uniform, then a is a constant coefficient and c is unity. In Lawful Relation 5, E is mechanical energy, f is frequency (that is, 1/T) and f_{int} is a positive quantity representing the lower bound on the frequency range over which a constancy of action H is realized, tailored to the magnitude of the wrist-pendulum system. The action constant H is an adiabatic invariant (roughly, a quantity that remains fixed during changes that do not move the system far from steady state) of the chemical-thermal-mechanical engine processes that power the cycles of a wrist-pendulum system. These processes are seated in the ensemble of muscle bundles of the radial and ulnar flexors together with their capillary blood supplies, their lymph supplies, their afferent and efferent nerve supplies, and the autonomic nerve supplies of their arterioles (cf. Bloch & Iberall, 1982). The important point to be underscored in the present context is that whereas Lawful Relation 5 guides the soft assembling of the chemical-thermal-mechanical engine processesthat is, the escapement-Lawful Relation 4 guides the soft assembling of the neural processes whose orderly dynamics compose the cycle duration. We could say, synonymously, that Lawful Relation 4 guides the soft assembling of a clock, as long as it is understood that a clock is simply anything whose states are ordered in a certain way (Bunge, 1977). The order in question is strict partial order, that is, asymmetric and transitive, so that no two states are concurrent (see Bunge, 1977, for the formal definition). To be a clock, whether of the soft molded neural kind or of the hard molded mechanical/electrical kind, requires moveable parts. By definition, a soft molded clock cannot consist of rigid parts in motion, like the hands and gears of an ordinary timepiece. Nor can it, by definition, consist of electrical signals guided along precisely defined paths. Its character more closely approximates that of a field. And its role more closely approximates that of a (local) frame or reference. Anything connectible with the clock can order its states by reference to the clock's states. Although it is customary to think of reference frames as rigid bodies, they need not be. A fluid field can be a frame of reference (Dehnen, 1970).

There are, in short, two functionally distinct assemblages (processes) supporting the periodic behavior of a wrist-pendulum system. [It is noteworthy that von Holst (1935/1973), some many years ago, drew a similar, though not identical, contrast. He proposed a functional distinction between (a) single central cells or populations of central cells that produce a periodic signal (now referred to as central pattern generators, see Selverston, 1980) and (b) populations of central cells, under this periodic control, that transmit impulses to the muscles.] Because both assemblages are soft molded (meaning, in part, that the resultant neural and metabolic flows are not constrained to precisely determined kinematic paths), their respective behaviors

straight ahead without looking at either wrist-pendulum system. At the start of the experimental session, a calibration trial was conducted for each hand so that an angular reference could be established for later use in the analyses. These trials consisted of a subject holding a pendulum first at 60° relative to his horizontally aligned forearms (and, therefore, to the ground plane) and then at 90° relative to his horizontally aligned forearms (and, therefore, perpendicular to the ground plane). A chiropractor's plastic goniometer was used in determining these angles.

Upon completion of these calibration trials, the subject was given verbal instructions regarding how he was to swing the pendulums. He was asked to comfortably grip the pendulum in such a way as to have complete control over the entire movement. He was also instructed to smoothly oscillate the pendulums forward and back using only the wrist joint (while continually keeping his forearms parallel to the ground plane). The subject was further directed to swing both pendulums with a single, common tempo either in phase or 180° out of phase (depending on the experimenter's instructions for each trial). Importantly, the subject was told that as he first started swinging, he was to search through a range of possible frequencies until he felt that he had settled on the most comfortable, stable tempo. He was given as long as he needed before each recorded trial to attain this preferred state (usually 5-15 s). When the subject felt that a comfortable tempo had been achieved, he verbally signaled the experimenter, who then started the recording process. Each recorded trial lasted for 15 s. After each trial had been recorded, the experimenter told the subject to stop swinging, and the next set of pendulums (according to a predetermined condition ordering) was placed into the subject's hands. The instruction set was repeated to each subject a number of times throughout the experiment, and each subject's behavior was closely monitored. The experiment lasted approximately 21/2 hr for each subject, including a 15-min break, which occurred halfway through the session.

There were 12 conditions. Conditions 1–6 involved Coupled Systems 1, 2, and 3. Conditions 7–12 involved Coupled Systems 4, 5, and 6. Within these 12 conditions, in 6 conditions the subjects swung the pendulums in phase (Conditions 1, 3, 5, 7, 9, and 11), and in 6 conditions the subjects swung the pendulums out of phase (Conditions 2, 4, 6, 8, 10, and 12).

Subjects were given 8 trials per condition. The 96 total trials were divided into 6 blocks of 16 trials each. Within each block, four conditions were performed four times each. The four conditions were composed of two in-phase and out-of-phase pairs, for example, Conditions 1 and 2 and Conditions 7 and 8. Trials within each block were given a random ordering. Across subjects the order of conditions was balanced in such a way that each in-phase/out-of-phase pair appeared an equal number of times in each of the six blocks.

Results

The digitized trials were first analyzed by the AngS and Vongraph waveform analysis programs at Haskins Laboratories. The peaks and valleys of the waveforms were "picked" using the criteria of the lowest and highest points of each cycle. Individual and mean peak-to-peak durations were used in calculation of the trial periods, period fluctuations, and Lag 1 autocorrelations. Mean valley-to-peak distances (in degrees) were used in the calculation of trial angular excursion and angular excursion fluctuation. The mean difference of one wrist pendulum's peak with respect to the other wrist pendulum's peak was used to calculate the trial deviation from intended phase. The data were further analyzed by the Haskins Laboratories' CPA program, which continuously measures the difference of phase between the two wrist pendulums on a sample-to-sample (200/s) basis, allowing the calculation of a fine-grained estimate of the fluctuations of phase. Exemplary waveforms and deviation from intended phase are shown in Figure 7.

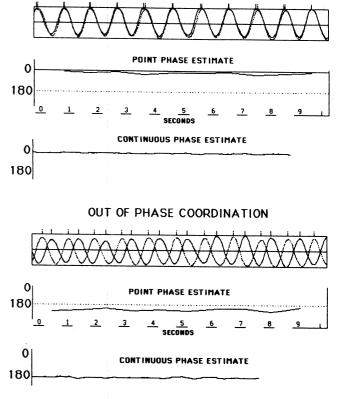


Figure 7. Examples of in-phase coordination and out-of-phase coordination together with point and continuous phase estimates.

In the analyses, the mean of the eight trials per condition was used for each observation. Analyses of the observed dependent variables of mean period, mean period fluctuation, and the mean Lag 1 autocorrelation will be presented first, followed by analyses of the calculated fluctuations of the motor and clock components. Next, analyses of the mean deviations from intended phase and phase difference fluctuations will be presented. All of the analyses performed (except where noted) were four-way analyses of variance (ANOVAS) with factors of Composition (same, different), Coupled Systems (System 1, System 2, System 3), Wrist (left, right), and Phase (in phase, out of phase). The Greenhouse-Geisser degrees of freedom adjustment for heterogeneity of covariance was used wherever applicable (Geisser & Greenhouse, 1959). Where the dependent variable is fluctuations, the standard deviation is the measure used in the analyses and in the presentation of the means. Variance rather than standard deviation (in deference to Wing & Kristofferson, 1973) is used in the figures.

Lag 1 Autocorrelation

The mean Lag 1 autocorrelations of period are presented in Figure 8. For all six coupled systems, the left and right Lag 1 values were negative. These negative values are a necessary condition for further analyzing the period variance into its motor and clock components: A cycle must compensate for its adjacent cycle's fluctuations. The four-way ANOVA revealed signifi-

COUPLED SYSTEM

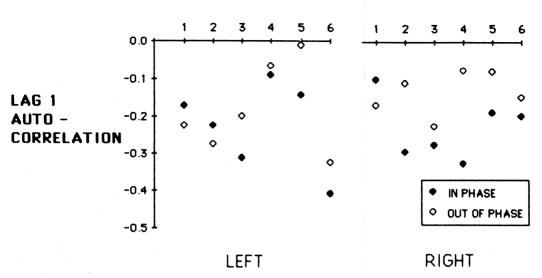


Figure 8. Mean Lag 1 autocorrelations of the left and right subsystems of the six coupled systems as a function of intended phase.

cant main effects of wrist, F(1, 3) = 10.55, $MS_e = 0.0012$, p < .05, and phase, F(1, 3) = 55.19, $MS_e = 0.0011$, p < .01. The effects speak to the difference in the cycle-to-cycle adjustment for fluctuations in the two wrists and in the two modes of coordination: More adjustment occurred in the left wrist (-.2407) than in the right wrist (-.2176), and more adjustments occurred with in-phase coordination (-.2542) than with out-of-phase coordination (-.2041). The composition, however, did not differentially affect the Lag 1 autocorrelation—same = -.2267; different = -.2316; F(1, 3) = 0.27, $MS_e = 0.0021$, p > .05. The Coupled System × Wrist interaction was significant, F(1.92, 5.76) = 8.36, $MS_e = 0.0028$, p < .05.

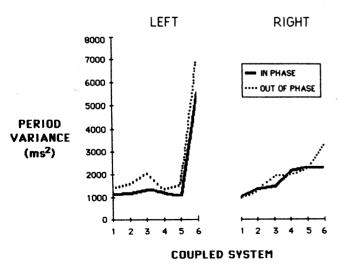


Figure 9. Mean variance of the period timing of the left and right subsystems of the six coupled systems as a function of intended phase.

Period Fluctuations

The four-way ANOVA yielded main effects of phase, F(1, 3) =29.39, $MS_c = 11.71$, p < .01; composition, F(1, 3) = 28.87, $MS_e = 88.98, p < .01$; and coupled system, F(1.95, 5.85) =27.25, $MS_e = 59.47$, p < .001. In-phase coordination showed less fluctuation in periodic timing (39.59) than did out-of-phase coordination (42.96). And different systems showed more overall period fluctuations (46.45) than same systems (36.10). Further there was a Composition × Coupled System × Wrist interaction, F(1.57, 4.27) = 20.79, $MS_e = 22.47$, p < .01. As is evident from inspection of Figure 9, the wrist that showed the most fluctuations was not the same wrist for the three systems of different composition. In contrast, the left wrist exhibited more fluctuations in all three of the same systems. The thrust of this finding is that with respect to fluctuations in periodic timing, the different systems were less homogeneous than the same systems. In sum, the period fluctuations were differentiated by both the mode of coordination and the composition of the coupled system.^{2,3} We will now proceed to the analyses of the period variance decomposed into its motor and clock components.

Motor and Clock Fluctuations Combined

The first requirement was an analysis that compared the two kinds of component fluctuations. A five-way ANOVA with fac-

² A basic finding of Wing and Kristofferson (1973; Wing, 1980) was not replicated here. They found that periodic fluctuations increased with an increase in period. An analysis of covariance with a covariate of period and factors of Composition, Coupled System, Intended Phase, and Wrist yielded neither an overall effect of the grand mean, F(1, 2) = 3.77, $MS_e = 2,776.4$, p > .05 nor an overall effect of the period, F(1, 2) = 1.25, $MS_e = 3,476.0$, p > .05.

³ The angular excursion means and fluctuations were also analyzed. No significant effects of phase and composition were found.

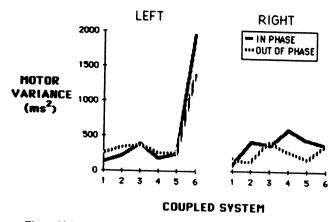


Figure 10. Mean motor variances of the left and right subsystems of the six coupled systems as a function of intended phase.

tors of Fluctuation Source (motor, clock), Composition (same, different), Coupled Systems (System 1, System 2, System 3), Wrist (left, right), and Phase (in phase, out of phase) was performed on the combined motor and clock fluctuation data. This analysis revealed a significant Fluctuation Source \times Composition \times Phase interaction, F(1, 3) = 10.64, $MS_e = 17.65$, p < .05. The significance of this effect is that the motor and clock fluctuations were differentially affected by phase and composition. Consequently, the sources of fluctuations can be analyzed separately.

Motor Fluctuations

A four-way ANOVA was performed on the motor component fluctuation data. Interestingly, there was no main effect of phase, F(1, 3) = 2.28, $MS_e = 19.60$, p > .05. Nor was there a significant interaction of phase and composition, F(1, 3) = 4.59, $MS_e = 20.42, p > .05$, as can be seen from inspection of Figure 10. In fact, in-phase coordination showed more fluctuations than out-of-phase coordination (in-phase = 19.28; out-ofphase = 17.91) opposite to the pattern evident in the periodic fluctuations depicted in Figure 9. There were significant main effects of composition, F(1, 3) = 4.00, $MS_e = 16.78$, p < .01, and coupled system, F(1.66, 4.99) = 9.30, $MS_e = 103.20$, p <.01. There were also significant interactions between coupled system and wrist, F(1, 1.65, 4.95) = 37.26, $MS_e = 9.44$, p <.01) and between composition, coupled system, and wrist, F(1.16, 3.48) = 19.51, $MS_e = 26.43$, p < .05. These effects show that in the motor fluctuations analysis, as well as in the periodic fluctuations analysis reported above, the same systems were more homogeneous in their fluctuations as a group than were the different systems. In addition, it showed that the left-wrist different systems were more heterogeneous than the right-wrist different systems.

Because the motor fluctuations were not affected by phase and because the motor fluctuations were affected by composition differently at the two wrists, one can conclude that the mechanism underlying motor component fluctuations (a) was unaffected by phase symmetry, (b) was sensitive to the composition of the coupled system, and (c) was defined locally at each wrist. This latter conclusion was corroborated in part by the absence of a significant correlation between left and right motor variances.

Clock Fluctuations

Clock component fluctuations were subjected to a four-way ANOVA. In contrast to motor fluctuations, clock fluctuations differed for the two modes of coordination: out-of-phase clock fluctuations (32.76) were greater than in-phase clock fluctuations (26.17), F(1, 3) = 19.70, $MS_e = 52.96$, p < .01. There were also significant main effects of composition, F(1, 3) = 13.77, $MS_e = 80.03$, p < .03, and coupled system, F(1.93, 5.78) = 5.78, $MS_e = 29.89$, p < .01. There was no significant Composition \times Coupled System \times Wrist interaction, F(1.43, 4.30) = .23, $MS_e = 52.46$, p > .05. Even though the different systems showed more clock fluctuations than same systems (same = 26.08; different = 32.85), this difference was constant across the two wrists (contrary to the case for the motor fluctuations). This can be seen in Figure 11.

Because the clock fluctuations were greater for systems in out-of-phase coordination and because clock fluctuations in both modes of coordination were affected in the same way by composition, it can be inferred that the mechanism underlying clock fluctuations was (a) affected by phase symmetry, (b) sensitive to the composition of the system, and (c) a unitary process in which the two wrists were treated as a single entity. This latter conclusion was partially corroborated by correlations between left- and right-wrist clock fluctuations computed over individual subject data, r(46) = .26, p < .06, and over coupled system means, r(10) = .71, p < .01.

Deviations From Phase Difference

A three-way ANOVA with factors of Composition, System, and Phase was performed on the difference between the actual phase relations and the intended phase relations (of 0° and 180°; see Figure 12). The analysis was conducted on Subjects 2, 3, and 4. The data on Subject 1 were lost prior to performance of the analysis. There was a significant effect of composition, F(1, 2) = 22.11, $MS_e = 0.001$, p < .05, with the different systems showing a significantly larger phase difference (same = 8.28°; different = 24.84°). The effect of intended phase was not significant, F(1, 2) = 0.82, $MS_e = 0.0003$, p > .05.

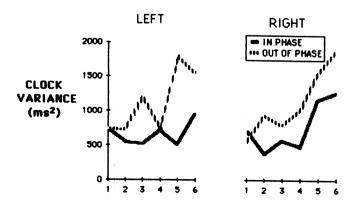


Figure 11. Mean clock variances of the left and right subsystems of the six coupled systems as a function of intended phase.

COUPLED SYSTEM

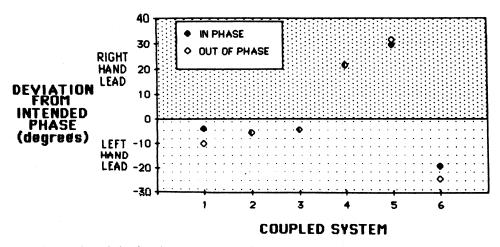


Figure 12. Deviation from intended phase as a function of coupled system and intended phase.

Phase Fluctuations

Fluctuations in continuous relative phase are shown in Figure 13. They were subjected to a three-way ANOVA with factors of Composition, System, and Phase. The ANOVA revealed only a significant effect of composition, F(1, 2) = 148.27, $MS_e = 0.978$, p < .01, with different systems showing a significantly larger amount of phase fluctuations (13.77) than the same systems (9.76). The effect of intended phase was not significant, F(1, 2) = 3.83, $MS_e = 27.08$, p > .05, although there was a tendency for out-of-phase coordination to show more phase fluctuations. The loss of the data of Subject 1 before the phase fluctuation analyses were done makes the analyses of phase fluctuations not strictly analagous to the analyses of other kinds of fluctuations above.

Discussion

When two wrist-pendulum systems are coupled to oscillate comfortably at the same tempo, the mean periodicity they assume is governed by Lawful Relation 4. More exactly, the mean periodic state is a function of $M^{.0625}L^{.5}$ or μ , where the single mass and length quantities for the coupled system are derived from the two mass and two length quantities of the left and right systems through equations anchored in the conservations. The relation between the mean periodic states of coupled wrist-pen-

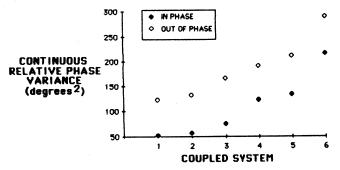


Figure 13. Mean continuous relative phase variance for each coupled system as a function of intended phase.

dulum systems, or virtual single systems (given that a coupled periodicity is determined by single quantities), and μ is unaffected by phase (in-phase vs. out-of-phase) and composition (same left/right systems vs. different left/right systems). Considerations of the properties of physical cooperativities and particular empirical observations (namely those of von Holst and Kelso) suggest, however, that phase and composition—though they may be immaterial to mean states—are strongly implicated in the fluctuations about mean states. Fluctuations in periodic timing were the focal concern of the present article. A secondary, but closely related, concern was the phase relation itself. The individual subject's intention was to oscillate the two wrist-pendulum systems at either a 0° phase difference or a 180° phase difference. The accuracy with which an intended phase difference was achieved was measured coarsely by mean phase and finely by fluctuations about the mean phase.

Composition and Fluctuations in Periodic Timing

Using the Wing-Kristofferson theory, we can parse the variance in periodic timing into that due to the chemical-thermal-mechanical engine processes that power the cyclic movement and that due to the partially ordered dynamical states (defining a clock) that function as a reference frame for these engine processes. The chemical-thermal-mechanical engine processes are local. They are largely confined to the region of the immediate source of chemical fuel from which the cycle's mechanical energy (mainly) derives. In simpler terms, the engine processes are localized in the musculature governing the limb segment in question. Where two wrist-pendulum systems are coupled, two local engine processes can be delimited. At each local site the

⁴ The more common measure of phase difference fluctuations is the point estimate measure. The phase difference forming the distribution is measured once per cycle rather than continuously throughout the cycle. Point estimate phase difference fluctuations were subjected to a three-way ANOVA with factors of Composition, System, and Phase. The ANOVA yielded no significant effects. This negative outcome is in agreement with the observations of Yamanishi, Kawato, and Suzuki (1979) for two finger tapping.

energy flow is conditioned, in part, by the local moment of inertia.

There are two points in the preceding comments on the engine processes or escapements that should be highlighted. First, they are tied to local quantities. Second, and related, the engine processes at one site are largely independent of the engine processes at another site. On these two points engine processes and clock processes seem to contrast sharply. Periodic timing is tied to distributed quantities. Specifically, periodic timing is tied to distributed quantities. Specifically, periodic timing is tied to μ_{ν} , which is derived lawfully from the magnitudes distributed over the left and right wrists. Consequently, clocking processes at one site (one wrist-pendulum system) should not be independent of clocking processes at another site (the other wrist-pendulum system).

Our goal now is to see how data and theory concur. Coupled Systems 1, 2, and 3 were distinguished on the S measure from Coupled Systems 4, 5, and 6. That measure, it will be recalled, is an approximate index of the compatability between the component wrist-pendulum systems and the virtual single system assembled from them. Variance in periodic timing was less for Coupled Systems 1, 2, and 3, whose S measures were identically zero, than for Coupled Systems 4, 5, and 6 whose S measures (averaged over subjects) were 0.258, 0.378, and -0.457, respectively. The composition of a virtual single system affected, therefore, the magnitude of the fluctuations about its mean periodic time. A local perspective on the composition variable reveals part of the reason for this effect. Because of the design of the experiment, Coupled Systems 1, 2, and 3 imposed the same demands, and Coupled Systems 4, 5, and 6 imposed different demands, upon the left and right wrist-pendulum systems. For Coupled System 6, for example, the μ_v/μ_l ratio (averaged over subjects) was 1.76, whereas the μ_v/μ_r ratio (averaged over subjects) was .98. In the assembling of the virtual single system, the left system was forced to depart from its preferred steady state much more than the right system and in a different direction than the right system. In Coupled System 6 the left system oscillated much more slowly, and the right system oscillated slightly more quickly than preferred. For Systems 4 and 5 the opposite was true. The ratios were $\mu_v/\mu_l = 0.93$ versus $\mu_v/\mu_r = 1.22$ and $\mu_v/\mu_l = 0.96$ versus $\mu_v/\mu_r = 1.50$. The inequality of these ratios refers to an inequality in the stresses incurred locally. Thus they should be manifest in motor variance and not in clock variance. In corroboration, a significant Wrist × Coupled System × Composition interaction was observed for the motor variance but not for the clock variance. Coupled systems whose components were the same distinguished from coupled systems whose components were different in that their motor fluctuations were more homogeneous. This reflects the equality of the μ_{ν}/μ_{l} and μ_v/μ_r ratios in the same couplings and the inequality of these ratios in the different couplings. The three-way interaction also points to a greater homogeneity of motor fluctuations in the right components than in the left components of different systems. This reflects the smaller contrast among right μ ratios (1.22, 1.50, 0.98) in Coupled Systems 4, 5, and 6 relative to the contrast among left μ ratios (0.93, 0.96, 1.76) in Coupled Systems 4, 5, and 6. One particularly noteworthy observation is the left versus right contrast for Coupled System 6 (see Figures 10 and 11). This contrast, markedly evident in the motor variance, is absent in the clock variance.

The preceding discussion highlights aspects of the linkage be-

tween a virtual single system's composition and fluctuations around its mean periodic state. Other aspects will be noted below. At this juncture we proceed to a consideration of the linkage between phase and fluctuations about the mean periodicity.

Phase Symmetry and Fluctuations in Periodic Timing

A phase difference close to 0° or 180° is established by the subject in accordance with the experimenter's instructions. It is one of the boundary conditions or constraints that the subject brings to bear, in the form of intentions, on natural laws (in particular those that govern oscillatory movements assembled and sustained by biological tissues and processes). It is to be supposed that the constraint for in-phase coordination differs from that for out-of-phase coordination. That is, although an underlying dynamics of periodic timing (a duration metric) is common to the two phase relations—for both of them the mean periodic time scales as μ_v —how these common dynamics are constrained differs between the two phase relations. In other words, the ordering of dynamic states that composes the clock for in-phase periodic timing is not the same ordering of dynamic states that composes the clock for out-of-phase periodic timing. The required strict partial ordering may be met more easily in phase than out of phase. If it is, then the stability or attractiveness distinction observed empirically (e.g., Kelso & Scholz, 1985; von Holst, 1935/1973) would be rationalized. This conjecture aside, if phase relation reduces to a constraint on clock states, then any differential consequences of phase should be restricted to clock variance. Let us see how the data fared in this respect.

To begin with, both the Lag 1 autocorrelation and the variance in periodic timing (s_T^2) proved to be sensitive to the inphase/out-of-phase distinction. That is, although an effect of phase was not found for the mean periodic times, it was found for two quantities that reflect the fluctuations about these mean states. The variance in periodic timing was greater (see Figure 9), and the average Lag 1 autocorrelation was smaller (see Figure 8), for out of phase than in phase. By Wing-Kristofferson's theory, of course, these measures are derivative rather than pure. They are what they are because of the underlying temporal imprecision in the (softly assembled) motor and clock processes. Each measure is jointly reflective of the two independent processes. So the question becomes: Did phase affect both processes or just one? More particularly, did phase affect just the clock, as the preceding arguments would imply? Inspection of Figures 10 and 11 suggests, and the analysis of variance confirms, that phase was a significant factor in clock variance but an insignificant factor in motor variance. Clock variance out of phase exceeded that in phase. The first conclusion we wish to draw from this observation is that the attractiveness contrast between in phase and out of phase noted by von Holst and Kelso is a contrast at the global level of the softly assembled clock rather than at the local level of the softly assembled engine or escapement processes. As conjectured above, clocks for inphase coordination are perhaps more stable, generally speaking, than clocks for out-of-phase coordination. A second conclusion we wish to draw is that a subject's intention to swing the wrist-pendulum systems in phase or out of phase was realized selectively as a constraint at the global clock level. This second

conclusion is corroborated by a further consideration of the effects of the composition variable.

Composition and Steady-State Phase Relation

By Scaling Relation 1 or, synonymously, by Lawful Relation 4, the preferred periodic timing of an individual or right wrist-pendulum system is determined by its μ value. An individual system's μ value, therefore, as noted in the introduction, is an index of the individual oscillatory system's steady state. As such, it provides a natural unit of measure for the system. Distance from the steady state can be expressed in terms of this unit. The left wrist-pendulum system of Coupled System 6, for example, was displaced a distance of $1.76\mu_l$, or 1.76 natural units, from its steady state. In general, distance from the steady state is given by $n\mu$, where n is a rational number.

Wrist-pendulum systems are individuated by their μ values. They are individuated, more correctly speaking, by their steady states. This individuality is merely compromised temporarily, not relinquished, when wrist-pendulum systems are assembled into a virtual single system. This subsistence of local steady states as local benchmarks is a most important fact. It was implicitly used in the motor variance analysis. The patterning of local motor variances was rationalized, in part, by differences in displacements from the steady state. That individual "atomisms" in a cooperativity maintain their individual integrity is a very general feature of physical, chemical, and biological phenomena. It was a fact well recognized by von Holst (1935/ 1973). He referred to the "maintenance tendency" of fin rhythm generators and meant by this expression the continuance of an individual generator's intrinsic or characteristic properties over the various locomotory organizations in which it participated.

The "maintenance tendency" is visible in cooperative states. The effect of displacements from the local steady states on motor variance is an example. Another is provided by the patterning of mean phase differences. The phase differences for Coupled Systems 4, 5, and 6, when the intended phase difference was 0°, were right leading by 21.24°, right leading by 29.16° and left leading by 19.80°, respectively. The phase differences for these same systems, when the intended phase difference was 180°, were right leading by 21.60°, right leading by 31.68°, and left leading by 25.20°, respectively. These phase differences are a manifestation of the "maintenance tendency": Right-leading differences are observed when $\mu_r < \mu_l$ (the right wrist-pendulum system oscillates faster in isolation than the left wrist-pendulum system), and left-leading differences are observed when $\mu_l < \mu_r$. In sum, the phase difference in the steady-state behavior of a coupled or virtual single system reflects the time-dependent steady states (natural periods) of its component systems. As noted in the introduction, von Holst (1935/1973) witnessed this phenomenon in the pectoral fin/dorsal fin phase relations of Labrus with medulla transection, and Stein (1973) has observed it in the swimmeret system of the crayfish.

To repeat, the phase relation a subject achieved and the phase relation a subject intended were not perfectly coincident. In particular, coupled systems whose components were different met the intended phase relations of 0° and 180° less well than the coupled systems whose components were the same. What

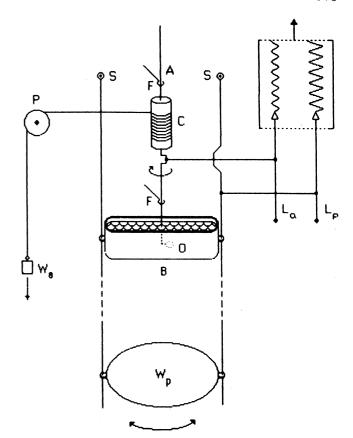


Figure 14. von Holst's model of a mechanical coupling of two oscillators through a viscous medium. (See text for details.)

does the steady state phase relation indicate? Why should it be sensitive to composition?

Establishment of Phase Differences in Nonbiological and Biological Coupled Oscillators

Consider the coupled oscillatory system devised by von Holst (1935/1973) and depicted in Figure 14. One oscillator is a pendulum suspended from S and composed of a weight Wp and a bath B; the bath contains a viscous mass (e.g., a syrup). The other oscillator is an axle A rotating around a fixed point F and driven by a thread that is wound around a cylinder C and linked to a weight Wa over a pulley P. At the axle's lower end is a spherical object 0 immersed in the viscous mass. The pendulum rhythmically moves the viscous mass to and fro. The frictional resistance acting on the sphere, together with the weight Wa, determine the inherent period of rotation of the axle. If the pendulum is immobile, then the axle rotates evenly, producing a sinusoidal oscillation of the recording lever La. If the pendulum also oscillates (recorded by the lever Lp), then the axle's periodic motion is modified by the motion of the viscous mass acting on the axle through the immersed sphere at the axle's lower end. The resultant periodicity depends upon the difference between the inherent periods of the two oscillators and the amplitude of the pendulum's motion. von Holst (1935/1973) was able to model the essential features of the rhythmic fin movements of Labrus with this apparatus, which mechanically couples two oscillators through a viscous medium.

When the amplitude of the pendulum's motion is suitably large, the pendulum's period becomes the axle's period. In von Holst's (1935/1973) terms, the two oscillators become absolutely coordinated at the tempo of the dominant oscillator. If the axle's inherent period is less than that of the pendulum, then it will oscillate at the pendulum's tempo but ahead of the pendulum's phase. If the axle's inherent period is greater than that of the pendulum, then it will oscillate at the pendulum's tempo but behind the pendulum's phase. At equilibrium, the phase at which the axle leads or lags the pendulum is determined by the overall force structure. More exactly, the balance among the different forces—the applied force, the gravitational force, and the viscous force—is achieved at a certain phase relation. A change in the axle's inherent period (for example, increasing the weight Wa), with the pendulum's size and amplitude of motion held constant, would result in a different configuration of the forces at equilibrium and a different phase relation. That is to say, the coupled phase difference (a) indexes a balance of forces (or a uniform distribution of conserved quantities) and (b) depends on the inherent periods of the component oscillators. Forces tend to distribute evenly when the inherent periods are the same. They distribute unevenly when the inherent periods are different. The phase difference "corrects," so to speak, for the uneveness. The more uneven (that is, the less uniform) is the distribution of forces, the larger is the "correction."

Oscillatory processes of biomechanical origin are distinguished from the oscillatory processes described in the preceding section by the fact that they are information based. The steady-state phase difference between two biomechanical oscillatory processes reflects, nonetheless, forces in balance. These claims require careful elaboration.

When two wrist-pendulum systems begin to oscillate together, there will be one mix of conservative and nonconservative forces centered in the left system and another mix centered in the right system. The force conditions at one site are not communicated in the ordinary physical sense to the other site. By "ordinary physical sense" we mean conservation transport, that is, the transport of the ordinary conserved quantitiesmass, momentum (linear, angular), and energy. These quantities are local in origin and remain local. The momentum and energy of one wrist-pendulum system are not transported to the other wrist-pendulum system. (Referring to the upper panel of Figure 1, if the person is swinging only one of the systems, then the other, motionless system will not be caused to move as a result; it will move rhythmically only if the person decides that it should.) In contrast, momentum and energy are transported between the two oscillators depicted in Figure 14. From this transport arises a balanced force configuration and, a fortiori, a specific phase difference.

Kugler and Turvey (1986) hypothesize that forces localized to different parts of the body induce field properties of a nonkinetic nature defined globally over the haptic system (Gibson, 1966). A local play of forces (at a wrist muscles-joint complex) stresses, in a patterned way, the mechanoreceptors ubiquitously embedded in the local (muscular, tendinous, ligamentous, vascular) tissues. Haptic stimulation "diffuses" over the nervous system from a local region to interact with other haptic "diffusions" of local origin and with the backdrop of haptic stimulation resulting from the body's disposition relative to gravity and surfaces of support. The consequent distribution of haptic stim-

ulation is fieldlike. But its continuously defined quantities are not orthodox kinetic quantities. They are quantities restricted to the dimensions of length and time. (The dimension of mass is suppressed.) In short, the haptic field's properties are kinematic and/or geometric and/or temporal, not kinetic. The continuously defined quantities of a kinetic field are mass based, by definition. The distribution of forces on the body's tissues define such a kinetic field (the dimensions of force are ML/T^2).

Following Gibson's (1966, 1979) treatment of information, it can be argued that the haptic field's properties are specific to the kinetic states of affairs that lawfully generated them. The haptic (neural) field is information about the kinetic (force) field. Kugler and Turvey (1986) wish to argue that properties of the haptic field will specify the attractor states of the underlying kinetic field. That is, the time-dependent balance of forces distributed over the left and right wrist-pendulum systems will be specified haptically. The ultimate significance of this line of argument is that where oscillators coupled mechanically through a viscous medium are forcefully impelled to their equilibrium phase difference, oscillators coupled intentionally through the nervous system are informationally guided to their steady-state phase difference. In both situations, however, the phase difference is lawfully grounded.

Implications of the Independence From Phase Symmetry of Deviations in Intended Phase

Returning to the data, the difference between the same and different coupled systems in terms of their departure from 0° and 180° is interpretable as a difference in the way the underlying forces configure at the steady state. For different systems the force distribution is less uniform. Consequently, the phase difference, that "corrects" for the unevenness, is larger. The size of the "correction," however, was statistically the same for the two intended phase relations of 0° and 180°. This insensitivity of the intended phase-actual phase difference to the intended phase relation seems to imply three things. The first implication is that the two intentional constraints—roughly, "left phase equals right phase" and "left phase equals right phase plus 180°"—harness the laws governing pendular, clocking behavior to produce tolerable phase differences rather than exact phase differences. The in-phase and out-of-phase intentions prescribe and maintain a general orientation of the oscillations of one wrist-pendulum system to the oscillations of the other. The particular orientation, that is, the particular phase difference actually exhibited $(0^{\circ} \pm x^{\circ})$ or $180^{\circ} \pm x^{\circ}$ in the comfort state, is determined by the uniformity of the force distribution which is linked, in turn, to the local steady states of the two wrist-pendulum systems. This observation suggests that an intended phase relation between wrist-pendulum systems (or biomechanical rhythm generators in general) of 0° or 180° will be realized perfectly only when the steady states of the two systems are identical. It also suggests that the two intentions of "phase" and "comfort" can be realized simultaneously only when the steady states of the two systems are identical.

From von Holst (1935/1973) we learn that many steady-state phase differences are achievable by the nervous system. The second implication of the same "correction" for in-phase and out-of-phase coordinations of a given coupled system bears on this multiplicity of phase differences. Apparently, if a given cou-

pling of rhythmic movements can operate comfortably and stably at a phase difference of x° , then it can operate comfortably and stably at $x^{\circ} + 180^{\circ}$. A given coupling always has two attractor states, exactly 180° apart. From the arguments presented above, of the two attractor states, the attractor state that is closer to in-phase coordination, closer to 0° difference, is preferred.

The third implication is closely related to the second. The balancing of conservative and nonconservative forces in the steady state coupling of two wrist-pendulum systems was indifferent to the in-phase/out-of-phase distinction. This fact points away from the locally assembled chemical-thermal-mechanical engine processes and toward the globally assembled clock as the location of the in-phase/out-of-phase distinction, To repeat the conclusion of above, a subject's intention to swing in phase or out of phase appears to be realized as a constraint on the strict partial ordering of dynamical states that define the globally assembled clock. By this conclusion, any tendencies for continuous relative phase to fluctuate more when the coordination was out-of-phase than in-phase would be attributable to the fluctuations of the clock. Clock fluctuations, it will be recalled, were greater under the out-of-phase constraint than under the in-phase constraint.

Summary: The Virtual Single System as an Up-Down Cooperativity

In concluding, we return to the notions of cooperative dynamics and virtual single systems. There are two forms of cooperativity (Shimizu & Haken, 1983). The two forms may be referred to as side-side cooperativity and up-down cooperativity. "Atomisms" or subsystems at the same level of analysis may interact directly with each other to fashion a cooperative state. In the case of two wrist-pendulum systems, the steady state periodicity and phase difference could derive from the oscillatory behavior of the left wrist-pendulum system directly affecting the oscillatory behavior of the right wrist-pendulum system, and vice versa. von Holst's (1935/1973) analysis of absolute and relative coordination of fin rhythms is in the spirit of side-side cooperativity as are most contemporary, coupled oscillator analyses of biological rhythms (e.g., Daan & Berde, 1978; Stein, 1976). The analysis of Kugler and Turvey (1986) and that presented here are closer to the spirit of up-down cooperativity. Atomisms or subsystems under a common field boundary condition, may give rise to a macroscopic, virtual quantity. This quantity is lawfully associated with a steady state. It is not the steady state of the individual atomisms, but it is the steady state onto which the individual atomisms relax. At the same time, however, the macroscopic quantity, derived as it is from the microscopic quantities, remains sensitive to them. There is a circular (macro-micro-macro-micro-...) causality (Kugler & Turvey, 1986; Yates, 1982).

In the case of the wrist-pendulum systems, μ_v is the macroscopic virtual quantity. It is lawfully fashioned from the mass and length quantities of the individual wrist-pendulum systems and lawfully related to a natural periodic time (a time-dependent steady state). This virtual property is at an intermediate level, between the boundary conditions or constraints (the subject's intentions) "above" and the left and right wrist-pendulum systems "below." The left and right wrist-pendulum systems, it is hypothesized, cooperate through their interactions with this

intermediate level. That is to say, they cooperate indirectly, in an up-down fashion.

This up-down perspective on cooperativity suggests that we should be able to describe the pendular, clocking mode of organization, defined over two wrist-pendulum systems, in terms of a closed circle of descending and ascending causal influences. The common periodic time of the two wrist-pendulum systems. and its indifference to the phase relation between them, are interpretable as consequences of the higher level to lower level causal link. The absence, in the clock variance analysis, of any interactions of phase with wrist or composition is interpretable as another consequence, albeit more subtle, of downward-projected influences. In sum, with respect to cycle duration, the interpretation from the indirect perspective is that the two wrist-pendulum systems behaved as a single unit, synchronized through a common set of partially ordered dynamical states that were lawfully determined by the macroscopic, virtual property μ_v . The correlation between mean left and mean right clock fluctuations (and the absence of such a correlation for motor fluctuations) lends support to this interpretation.

Ascending influences can similarly be identified. These effects, caused by the atomisms (the component wrist-pendulum systems), were not on μ_{ν} but on the duration metric that it lawfully engendered. Larger differences between the steady states of the component wrist-pendulum systems, relative to the steady state of the virtual single system, led to (a) larger clock fluctuations and (b) larger fluctuations in continuous relative phase. The enduring character of the component systems under the descending influence of the macroscopic, virtual property was evident. Motor variance was patterned by differences in the degrees to which components departed from their preferred steady states. Also, the magnitudes and directions of differences in phase were determined by the steady states of the component systems.

In sum, the pendular, clocking mode of movement organization is interpretable as an up-down, or synergic (Haken, 1983) cooperativity. The cooperative state of in-phase pendular, clocking behavior and the cooperative state of out-of-phase pendular, clocking behavior are basically the same in all respects but one: The softly assembled, partially ordered dynamical states that provide the reference frame for clocking are more stable (fluctuate less) in phase than out of phase.

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Appendix

Calculating the Virtual Single-System Magnitudes

A Simple Pendulum Equivalent of a Single Wrist-Pendulum System

An individual wrist-pendulum system is a compound pendulum replaceable by an equivalent simple pendulum. The length of an equivalent simple system can be derived as follows, through repeated use of Steiner's parallel-axis theorem (e.g., see Feynman, Leighton, & Sands, 1973):

1. The rods used in the construction of the pendulums were of uniform density and cross-sectional area. The constant product of density and area was found to be 0.248 kgm⁻¹. Consequently, the mass of any given rod was

$$M_{\rm rod} = (0.248)L,$$

where L was a rod's length in meters.

2. Let the clenched hand be approximated by a disc of radius a (Figure A1, Panel a), where 2a is the width of the hand measured from the second to the fifth carpometacarpal joints (Figure A1, Panel b).

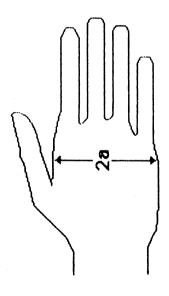




Figure A1. Top panel shows the distance between the second and fifth carpometacarpal joints. Bottom panel approximates the clenched hand by a disc.

- 3. Let b be the distance from the distal head of the radius bone in the forearm to the vertical axis of a rod held in the hand so that the top of the rod is flush with the top of the hand (Figure A2). The distances a and b will vary with the individual subject. For the 4 participants in the experiment the length of a was 0.0413 m, 0.0432 m, 0.0445 m, and 0.0413 m, respectively; and b was 0.0914 m, 0.0940 m, 0.0953 m, and 0.0931 m.
- 4. The center of mass of a rod lies at a distance of [(L/2) a] from the center of the hand which is at a distance b (on a line parallel to the ground plane) from the point of rotation O in the radio-ulnar joint (Figure A2). These distances are schematized in Figure A3. The distance from O to the center of mass of the rod is given by

$$Q = [b^2 + ((L/2) - a^2]^{1/2}$$

By the parallel-axis theorem the inertia of the rod about O is given by

$$I_{\rm rod} = M_{\rm rod}Q^2 + (M_{\rm rod}L^2)/12.$$

5. The added masses were discs attached to a bolt through the rod. The discs were evenly balanced on either side of the rod, allowing a construal of the discs as a single thick disc of radius $R=0.03~\mathrm{m}$ at a distance c from the lower end of the rod. The center of mass of this "disc" is a distance P from the point of rotation O as shown in Figure A4

$$P = [b^2 + (L - a - c)^2]^{1/2}.$$

By the parallel-axis theorem, the inertia of a disc about O is

$$I_{\rm disc} = M_{\rm disc}P^2 + (M_{\rm disc}R^2)/2$$

6. According to Dempster's (1955) and Bernstein's (1967) calculations, the mass of a hand is approximately 0.006% of a person's total mass. For the participants in the experiment, the hand mass was 0.435 kg for Subject 1, 0.468 kg for Subject 2, 0.490 kg for Subject 3, and

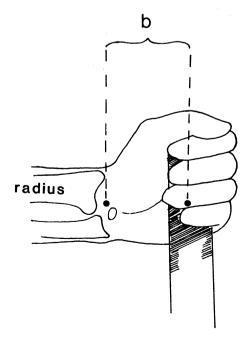


Figure A2. Distance b from the vertical axis of a hand-held rod to the axis of rotation 0 in the radio-ulnar joint.

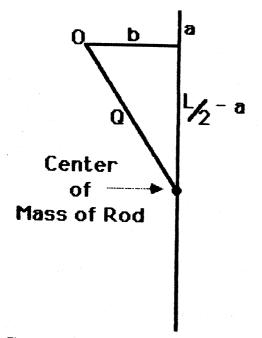


Figure A3. Schematic of rod giving dimensions relative to center of mass and axis of rotation.

0.435 kg for Subject 4. The distance of the hand's center of mass from O is b (Figure A2). Consequently, the inertia of a closed fist (conceptualized as a disc, Figure A1, Panel a) is, by the parallel-axis theorem,

$$I_{\text{hand}} = M_{\text{hand}}b^2 + (M_{\text{hand}}a^2)/2.$$

7. By Huygen's law the length of an equivalent simple pendulum is given by the ratio of a compound system's total moment of inertia to its total static moment. Determining the latter quantity requires the distance from the point of rotation O to the compound pendulum's center of mass (CM). The CM of the compound pendulum is at a distance of $L_{\rm CM}$ from the handle end of a rod as given by

$$L_{\rm CM} = \frac{M_{\rm rod}(L/2) + M_{\rm disc}(L-c) + M_{\rm hand}a}{M_{\rm rod} + M_{\rm disc} + M_{\rm hand}}.$$

The distance h from the point of rotation O to the compound pendulum's center of mass can then be calculated from

$$h = [L_{\rm CM} - a)^2 + b^2]^{1/2}$$

8. With h known, the length of an equivalent simple pendulum $L_{\rm e}$ is obtained from

$$L_e = \frac{I_{\text{rod}} + I_{\text{disc}} + I_{\text{hand}}}{(M_{\text{rod}} + M_{\text{disc}} + M_{\text{hand}})h}.$$

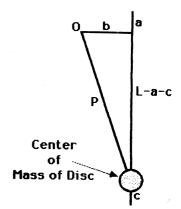


Figure A4. Schematic of added masses (construed as a disc) giving dimensions relative to center of mass and axis of rotation.

9. The mass of the equivalent simple pendulum can be considered as the summed masses of rod, disc, and hand concentrated at a point a distance of L_e from the point of suspension.

A Simple Pendulum Equivalent of a Double Wrist-Pendulum System

A compound pendulum is usefully thought of as two or more individual pendulums, rigidly coupled so as to restrict the systems' independent motions to a single common natural period. Figure 3 applies the concept of a compound pendulum organization to the double wrist-pendulum system: Figure 3a identifies a pair of simple pendulum equivalents of independent (isolated) wrist-pendulum systems and their associated natural periods; Figure 3b identifies the pair of pendulums coupled to form a double wrist-pendulum system with a single common period; and, Figure 3c identifies the equivalent "virtual" simple pendulum derived from Huygens' law. The mass of a virtual system, M_v , is the sum of the masses of the two component systems, and the length of the virtual system, L_v , is given as

$$L_v = \frac{M_1 L_1^2 + M_2 L_2^2}{M_1 L_1 + M_2 L_2},$$

where M_1 and M_2 are the two individual (equivalent simple pendulum) system masses, and L_1 and L_2 are the two individual (equivalent simple pendulum) lengths.

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